Text Classification

CS 5525: Foundations of Speech and Language Processing

https://shocheen.github.io/cse-5525-fall-2025/

THE OHIO STATE UNIVERSITY

Sachin Kumar (kumar.1145@osu.edu)

Slide Credits: Yulia Tsvetkov

Logistics

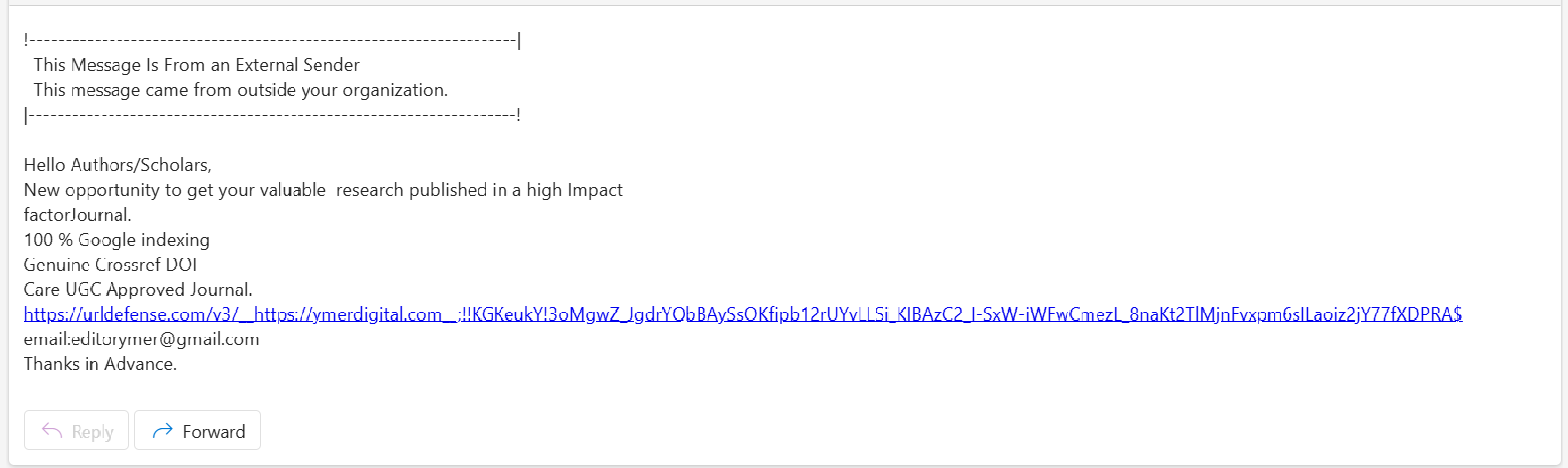
- Attendance.

- HW1 was released on Wednesday. Due Sept 10 (11.59pm)
  - It is based on today's and next week's lectures.
  - Have you reviewed it yet?
  - Post questions on Canvas

- Have you gained access to OSC?
  - Details on Canvas.

Text Classification

Is this spam?



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This Message Is From an External Sender
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Hello Authors/Scholars,
New opportunity to get your valuable research published in a high Impact
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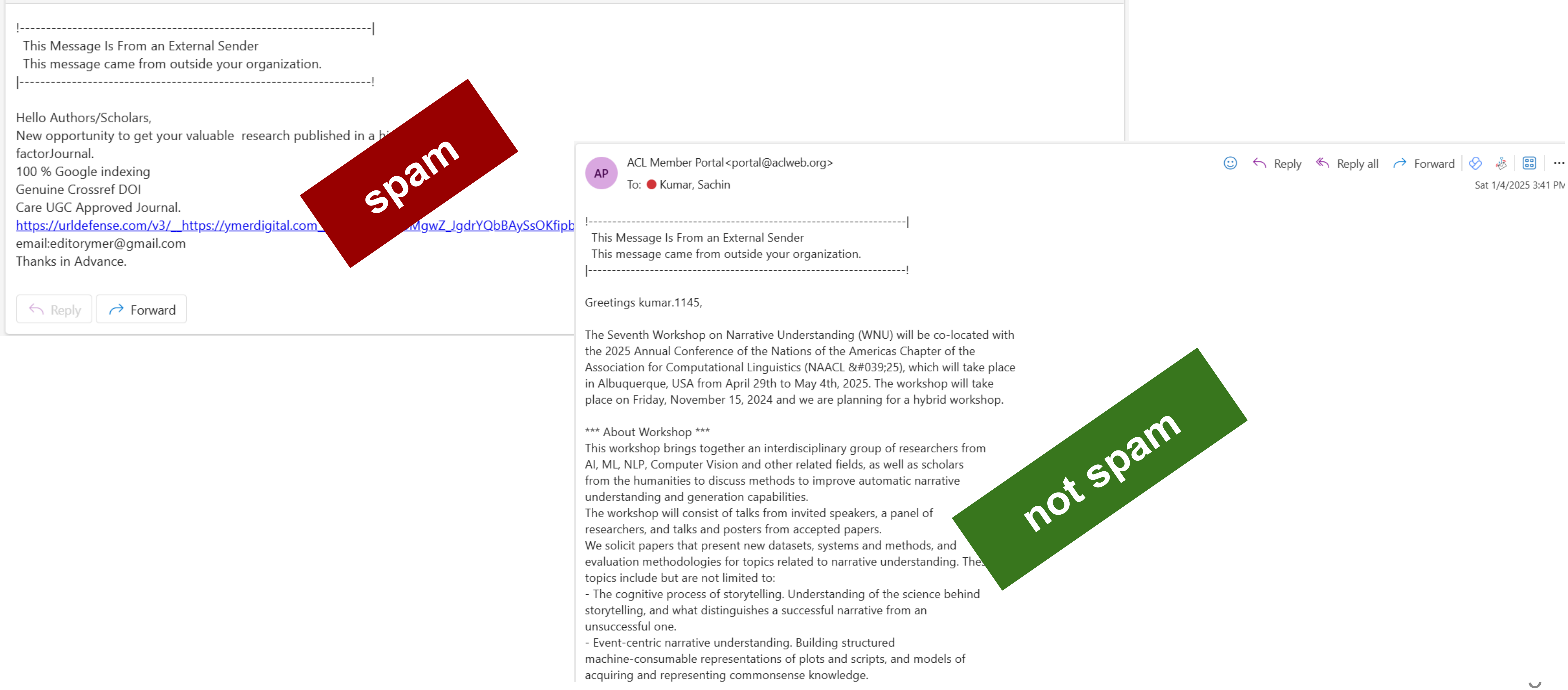
email:editorymer@gmail.com
Thanks in Advance.

Spam classification



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ACL Member Portal<portal@aclweb.org>
To: ● Kumar, Sachin
Sat 1/4/2025 3:41 PM

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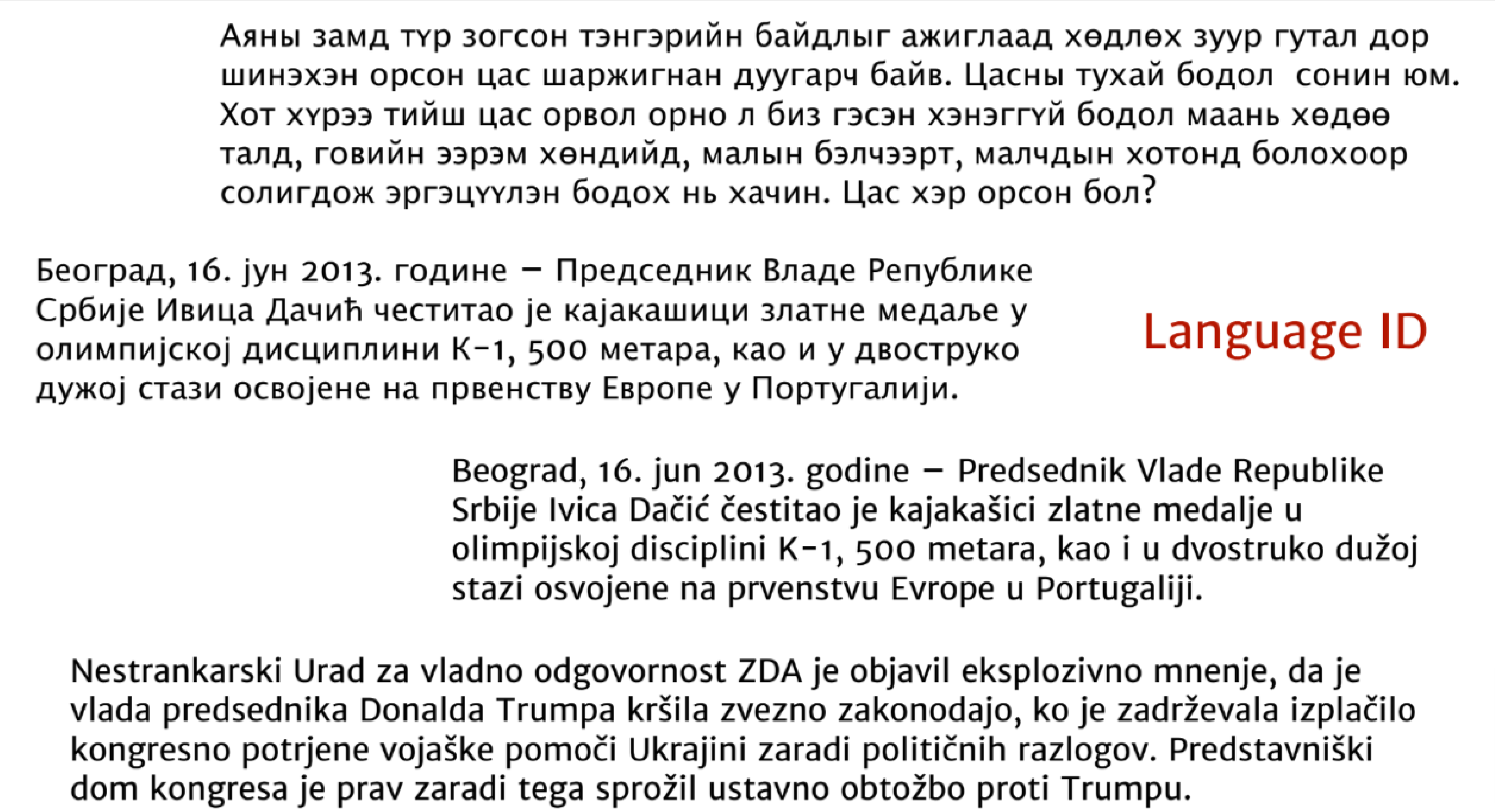
Greetings kumar.1145,

The Seventh Workshop on Narrative Understanding (WNU) will be co-located with the 2025 Annual Conference of the Nations of the Americas Chapter of the Association for Computational Linguistics (NAACL &#039;25), which will take place in Albuquerque, USA from April 29th to May 4th, 2025. The workshop will take place on Friday, November 15, 2024 and we are planning for a hybrid workshop.

*** About Workshop ***
This workshop brings together an interdisciplinary group of researchers from AI, ML, NLP, Computer Vision and other related fields, as well as scholars from the humanities to discuss methods to improve automatic narrative understanding and generation capabilities.
The workshop will consist of talks from invited speakers, a panel of researchers, and talks and posters from accepted papers.
We solicit papers that present new datasets, systems and methods, and evaluation methodologies for topics related to narrative understanding. The topics include but are not limited to:
- The cognitive process of storytelling. Understanding of the science behind storytelling, and what distinguishes a successful narrative from an unsuccessful one.
- Event-centric narrative understanding. Building structured machine-consumable representations of plots and scripts, and models of acquiring and representing commonsense knowledge.

Language ID



Аяны замд түр зогсон тэнгэрийн байдлыг ажиглаад хөдлөх зуур гутал дор шинэхэн орсон цас шаржигнан дуугарч байв. Цасны тухай бодол сонин юм. Хот хүрээ тийш цас орвол орно л биз гэсэн хэнэггүй бодол маань хөдөө талд, говийн ээрэм хөндийд, малын бэлчээрт, малчдын хотонд болохоор солигдож эргэцүүлэн бодох нь хачин. Цас хэр орсон бол?

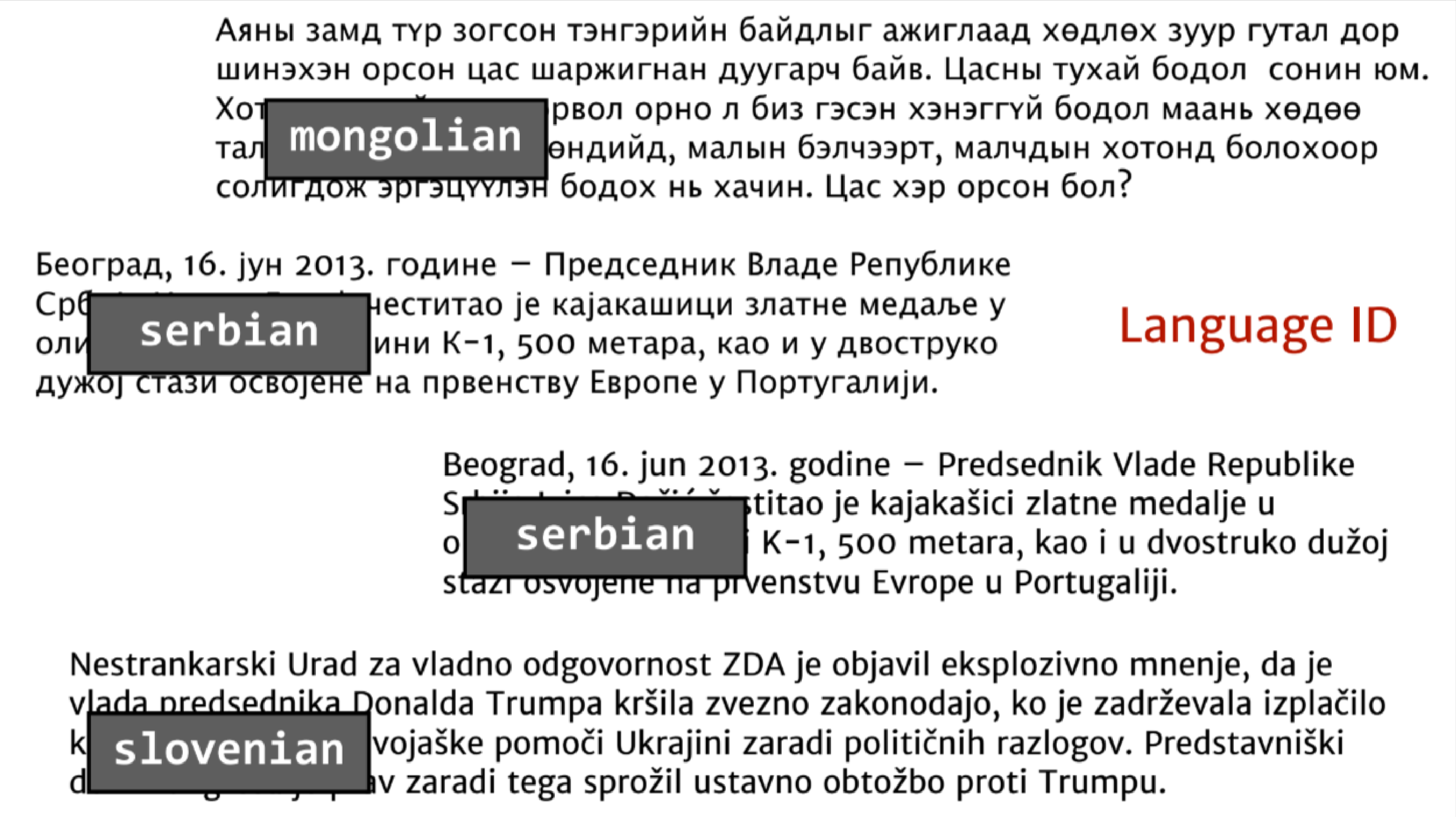
Београд, 16. јун 2013. године – Председник Владе Републике Србије Ивица Дачић честитао је кајакашици златне медаље у олимпијској дисциплини К-1, 500 метара, као и у двоструко дужој стази освојене на првенству Европе у Португалији.

Language ID

Beograd, 16. jun 2013. godine – Predsednik Vlade Republike Srbije Ivica Dačić čestitao je kajakašici zlatne medalje u olimpijskoj disciplini K-1, 500 metara, kao i u dvostruko dužoj stazi osvojene na prvenstvu Evrope u Portugaliji.

Nestrankarski Urad za vladno odgovornost ZDA je objavil eksplozivno mnenje, da je vlada predsednika Donalda Trumpa kršila zvezno zakonodajo, ko je zadrževala izplačilo kongresno potrjene vojaške pomoči Ukrajini zaradi političnih razlogov. Predstavniški dom kongresa je prav zaradi tega sprožil ustavno obtožbo proti Trumpu.

Language ID



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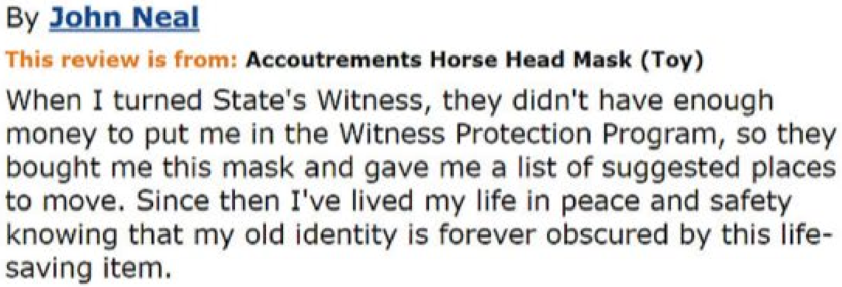
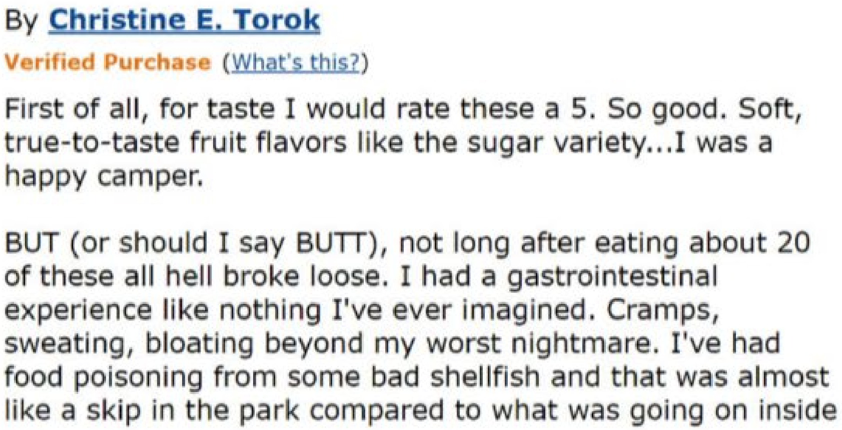
Sentiment analysis



By **John Neal**

This review is from: **Accoutrements Horse Head Mask (Toy)**

When I turned State's Witness, they didn't have enough money to put me in the Witness Protection Program, so they bought me this mask and gave me a list of suggested places to move. Since then I've lived my life in peace and safety knowing that my old identity is forever obscured by this life-saving item.



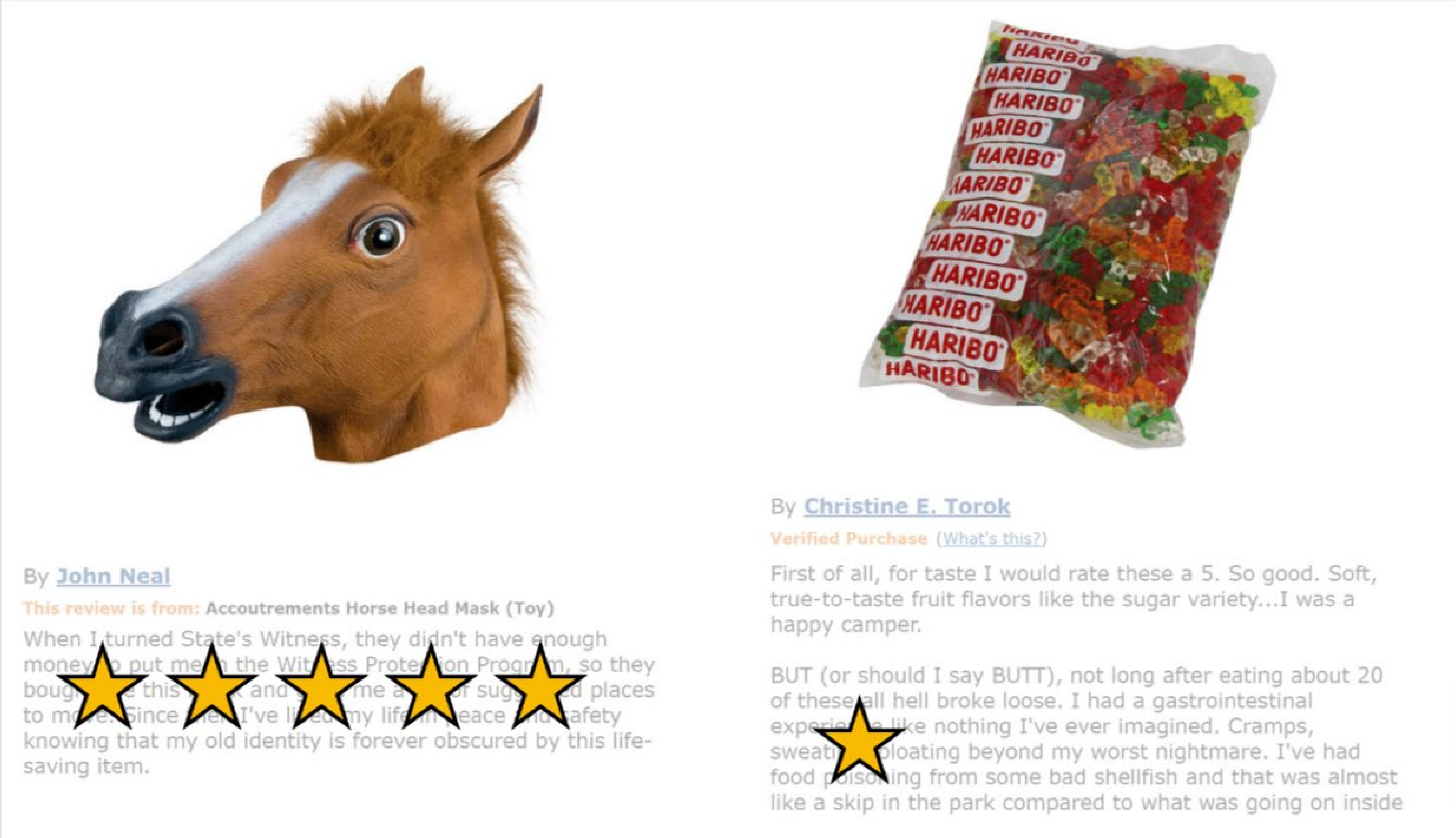
By **Christine E. Torok**

Verified Purchase (What's this?)

First of all, for taste I would rate these a 5. So good. Soft, true-to-taste fruit flavors like the sugar variety...I was a happy camper.

BUT (or should I say BUTT), not long after eating about 20 of these all hell broke loose. I had a gastrointestinal experience like nothing I've ever imagined. Cramps, sweating, bloating beyond my worst nightmare. I've had food poisoning from some bad shellfish and that was almost like a skip in the park compared to what was going on inside

Sentiment analysis



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★

Topic classification

MEDLINE Article

Available online at www.sciencedirect.com

Brain and Cognition

Syntactic frame and verb bias in aphasia: Plausibility judgments of undergoer-subject sentences

Susanne Gahl,[a] Lise Menn,[b] Gail Ramsberger,[b] Daniel S. Jurafsky,[b] Elizabeth Elder,[b] Molly Rewega,[a] and L. Holland Audrey[a]

MeSH Subject Category Hierarchy

- Antagonists and Inhibitors
- Blood Supply
- Chemistry
- Drug Therapy
- Embryology
- Epidemiology
- …

Authorship attribution: is the author male or female?

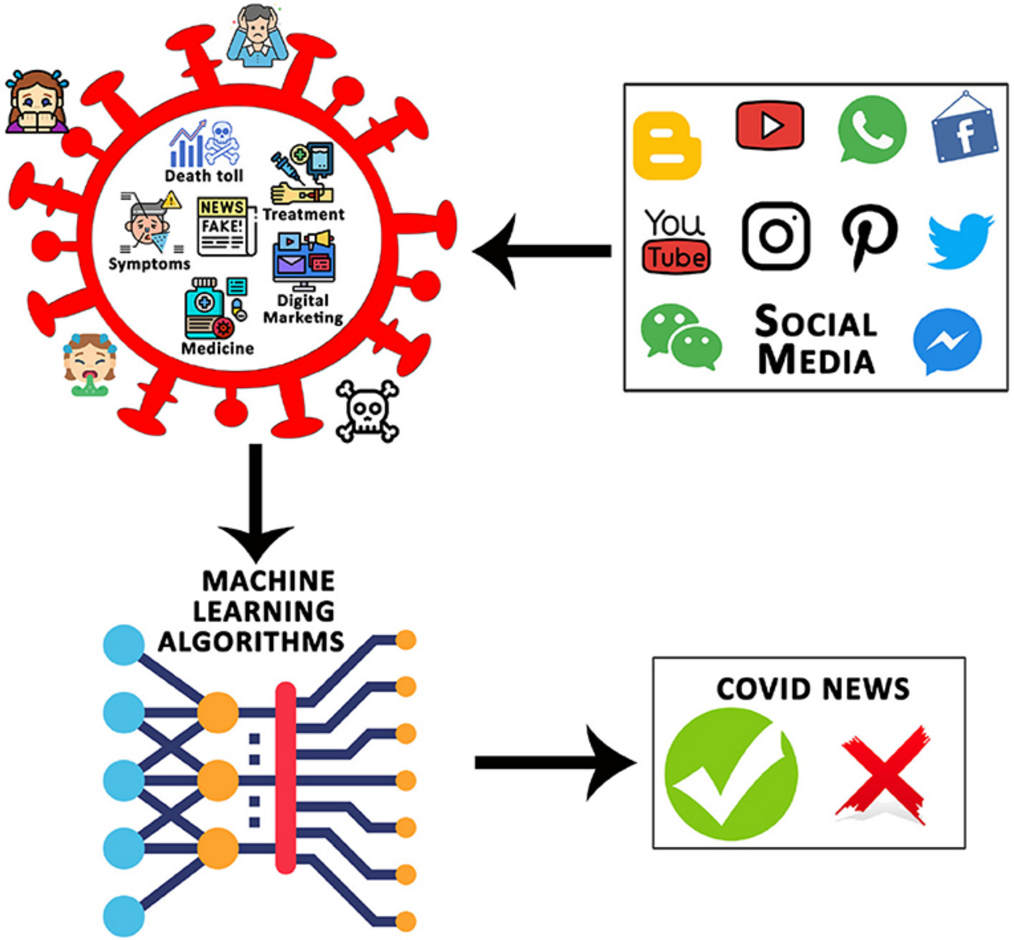
By 1925 Vietnam was divided into three parts under French colonial rule. The southern region embracing Saigon and the Mekong delta was the colony Cochin-China; the central area with its imperial capital at Hue was the protectorate of Annam.

Clara never failed to be astonished by the extraordinary felicity of her own name. She found it hard to trust herself to the mercy of fate, which had managed over the years to convert her greatest shame into one of the greatest assets…

S. Argamon, M. Koppel, J. Fine, A. R. Shimoni, 2003. “Gender, Genre, and Writing Style in Formal Written Texts,” Text, volume 23, number 3, pp. 321–346

Fact verification: trustworthy or fake?

Have Covid19?
Drink bleach!



Detecting COVID-19-Related Fake News Using Feature Extraction

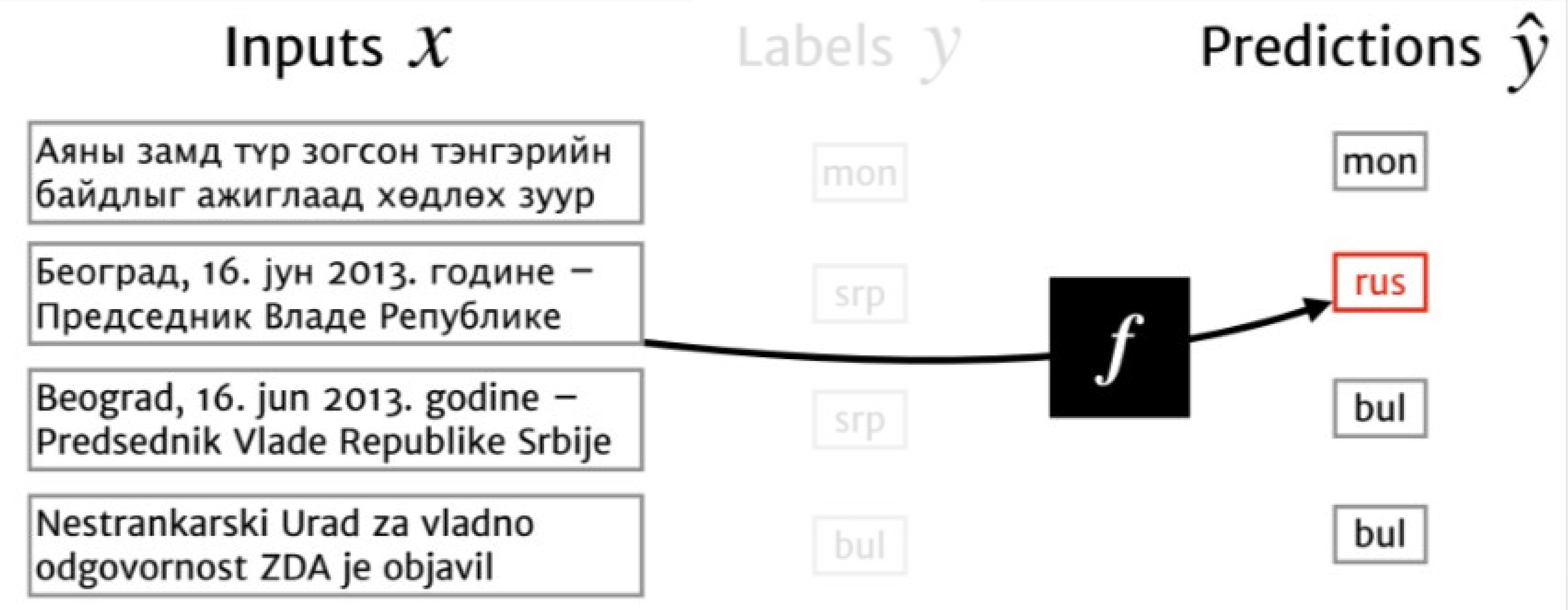
Suleman Khan, Saqib Hakak, N. Deepa, B. Prabadevi, Kapal Dev and Silvia Trelova

Text classification

- We might want to categorize the content of the text:
  - Spam detection (binary classification: spam/not spam)
  - Sentiment analysis (binary or multiway)
    - movie, restaurant, product reviews (pos/neg, or 1-5 stars)
    - political argument (pro/con, or pro/con/neutral)
    - Topic classification (multiway: sport/finance/travel/etc)
  - Language Identification (multiway: languages, language families)
  - ...
- Or we might want to categorize the author of the text (authorship attribution)
  - Human- or machine generated?
  - Native language identification (e.g., to tailor language tutoring)
  - Diagnosis of disease (psychiatric or cognitive impairments)
  - Identification of gender, dialect, educational background, political orientation (e.g., in forensics [legal matters], advertising/marketing, campaigning, disinformation)
  - ...

Text classification

Goal: create a function *f* that makes a prediction $\hat{y}$ given an input *x*



Inputs x
Labels y
Predictions $\hat{y}$
Аяны замд түр зогсон тэнгэрийн байдлыг ажиглаад хөдлөх зуур
Београд, 16. јун 2013. године – Председник Владе Републике
Beograd, 16. jun 2013. godine – Predsednik Vlade Republike Srbije
Nestrankarski Urad za vladno odgovornost ZDA je objavil
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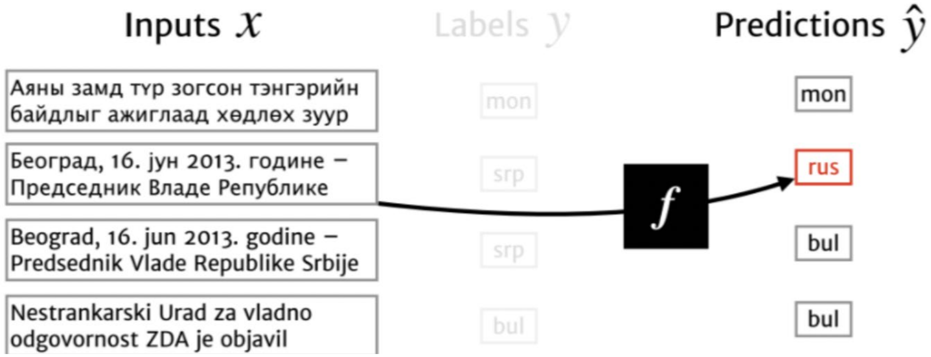
Today's plan:



Inputs x	Labels y	Predictions $\hat{y}$
Аяны замд тур зогсон тэнгэрийн байдлыг ажиглаад хөдлөх зуур	mon	mon
Београд, 16. јун 2013. године – Председник Владе Републике	srp	rus
Beograd, 16. jun 2013. godine – Predsednik Vlade Republike Srbije	srp	bul
Nestrankarski Urad za vladno odgovornost ZDA je objavil	bul	bul

1. How do we evaluate our function *f?*
   - (Keyword for this section: … evaluation)

2. How do we "digest" text into a form usable by a function?
   - (Keywords for this section: features, feature extraction, feature selection, representations)

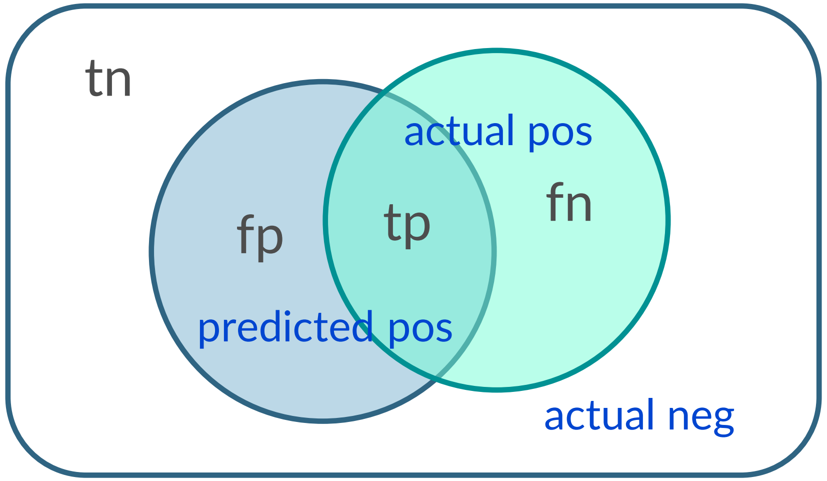
3. What kinds of strategies might we use to create our function *f?*
   - (Keyword for this section: models)

How do we evaluate our function *f*?

Classification evaluation

- Contingency table: model's predictions are compared to the correct results
  - a.k.a. confusion matrix

	actual pos	actual neg
predicted pos	true positive (tp)	false positive (fp)
predicted neg	false negative (fn)	true negative (tn)



Classification evaluation

- A simple (and often naïve) measure of performance: **accuracy**
    - the proportion of items the system got right

$$\text{accuracy} = \frac{tp+tn}{tp+fp+tn+fn}$$

	actual pos	actual neg
predicted pos	true positive (tp)	false positive (fp)
predicted neg	false negative (fn)	true negative (tn)

Why is accuracy “naïve”?

- Let’s say your test set has 50 positive examples and 50 negative examples.
    - Your model predicts them all correctly – what’s the accuracy?
        - 100%!
    - Your model predicts them all as positive – what’s the accuracy?
        - 50%

- Let’s say your test set has 90 positive examples and 10 negative example
    - Your model predicts them all as positive – what’s the accuracy?

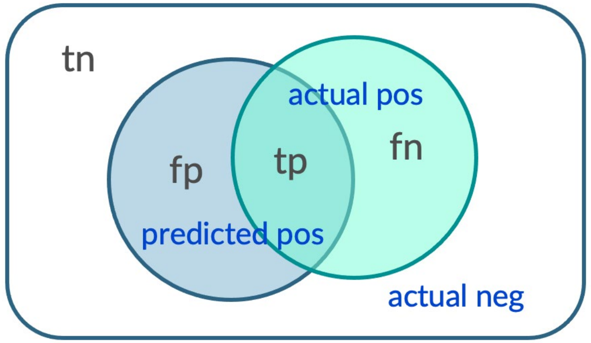
Better metrics: Precision and Recall

- Borrowing from Information Retrieval, empirical NLP systems are usually evaluated using the notions of precision and recall

Classification evaluation

- Precision (P) is the proportion of the selected items that the system got right in the case of text categorization
  - it is the % of documents classified as "positive" by the system which are indeed "positive" documents
- Reported per class or average

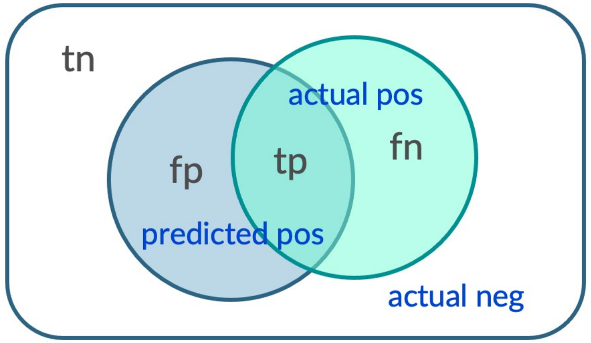
$$\text{precision} = \frac{\textit{true positives}}{\textit{true positives} + \textit{false positives}} = \frac{tp}{tp + fp}$$



Classification evaluation

- Recall (R) is the proportion of actual items that the system selected in the case of text categorization
  - it is the % of the “positive” documents which were actually classified as “positive” by the system
- Reported per class or average

$$\text{recall} = \frac{true\ positives}{true\ positives + false\ negatives} = \frac{tp}{tp + fn}$$



Classification evaluation

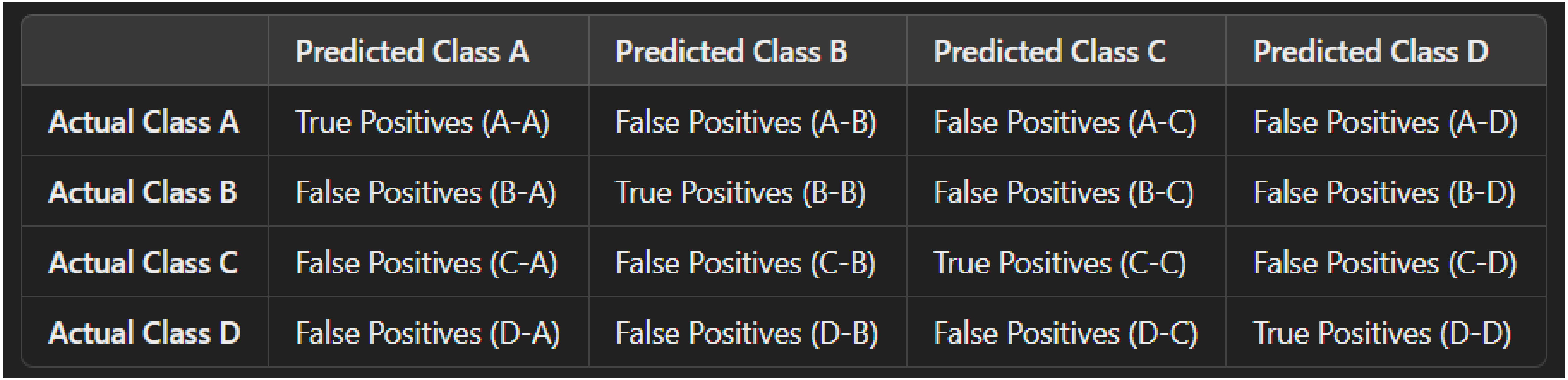
- There is often a trade-off in precision and recall
  - typically: the higher the precision the lower the recall

- It is convenient to combine P and R into a single measure
  - one possible way to do that is F measure

$$F_\beta = \frac{(\beta^2+1)PR}{\beta^2 P+R} \quad \text{for } \beta=1,\ F_1 = \frac{2PR}{P+R}$$

Multi-class classification

- For n-class classification, create an n x n confusion matrix



	Predicted Class A	Predicted Class B	Predicted Class C	Predicted Class D
Actual Class A	True Positives (A-A)	False Positives (A-B)	False Positives (A-C)	False Positives (A-D)
Actual Class B	False Positives (B-A)	True Positives (B-B)	False Positives (B-C)	False Positives (B-D)
Actual Class C	False Positives (C-A)	False Positives (C-B)	True Positives (C-C)	False Positives (C-D)
Actual Class D	False Positives (D-A)	False Positives (D-B)	False Positives (D-C)	True Positives (D-D)

Micro- vs. macro-averaging

If we have more than one class, how do we combine multiple performance measures into one quantity?

- Macroaveraging
  - Take the average of F1 score over all classes (all classes are weighted equally)
- Microaveraging
  - Compute one precision considering all classes and one recall considering all classes (all instances are weighted equally)

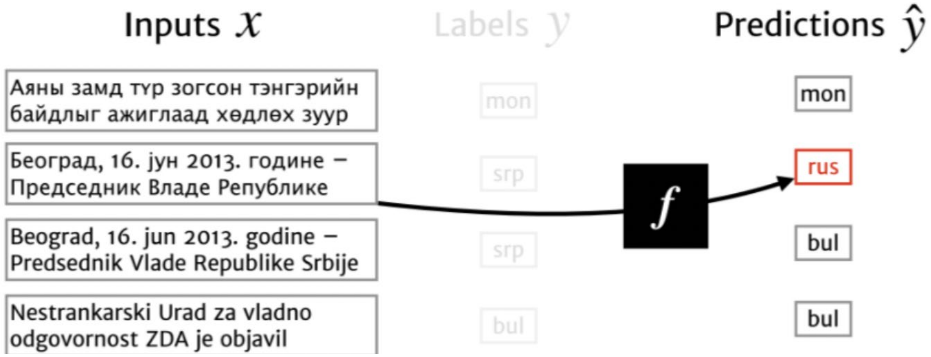
To summarize

- Accuracy is commonly used but can be deceiving

- Precision, recall, and F1 scores are better metrics. Precision and recall often have a trade-off

- In multi-class classification, you can report macro or micro F measures (or precision or recall)

Today’s plan: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How do we “digest” text into a form usable by a function?

Classification: features (measurements)

- Perform measurements and obtain features

4.2, 212, 3.4, 1332

↓ ↓ ↓ ↓

diameter, weight, softness, color

5.2, 315, 5.7, 4567

↓ ↓ ↓ ↓

diameter, weight, softness, color

Text classification – feature extraction

What can we measure over text?

Consider this movie review:

I love this movie! It's sweet, but with satirical humor. The dialogue is great, and the adventure scenes are fun… It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it just to about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it before.

Text classification – feature extraction

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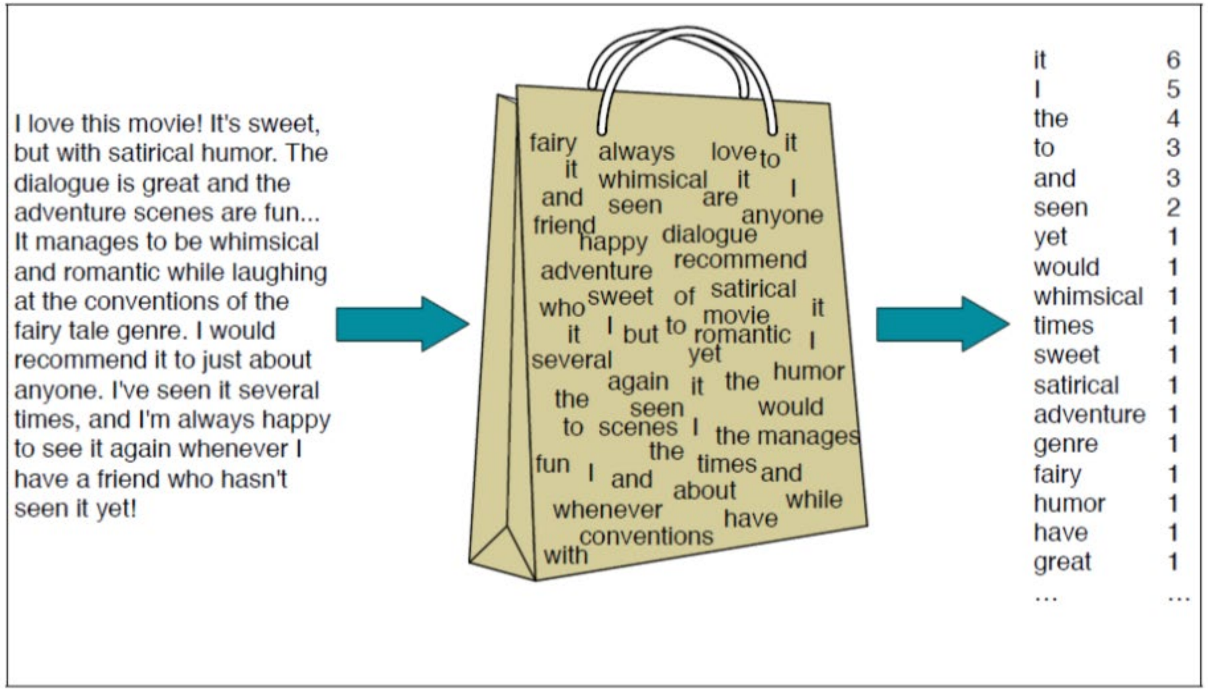
Bag-of-Words (BOW)

Given a document d (e.g., a movie review) – how to represent d ?



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it	6
I	5
the	4
to	3
and	3
seen	2
yet	1
would	1
whimsical	1
times	1
sweet	1
satirical	1
adventure	1
genre	1
fairy	1
humor	1
have	1
great	1
...	...

Figure from J&M 3rd ed. draft, sec 7.1

BOW feature extraction, independence assumption

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(almost) the entire lexicon

word	count	relative frequency
love	10	0.0007
great	…	
recommend		
laugh		
happy		
…		
several		
boring		
…		

Types of textual features beyond BOW

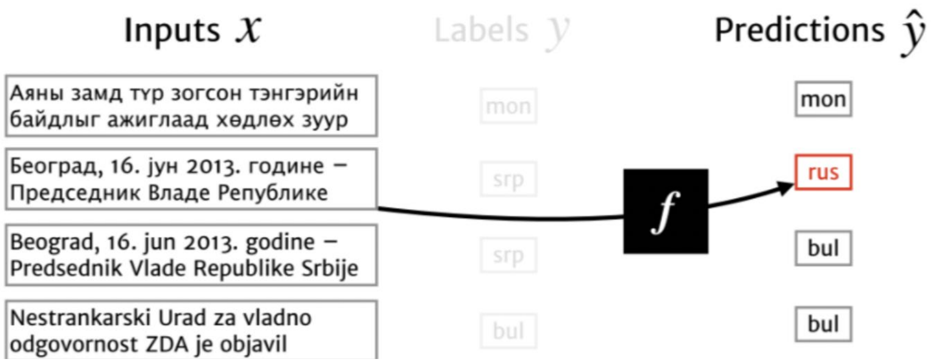
- Words
  - content words, stop-words
  - punctuation? tokenization? lemmatization? lowercase?
- Word sequences
  - bigrams, trigrams, n-grams
- Grammatical structure, sentence parse tree
- Words' part-of-speech
- Word vectors
- …

Summary: Possible representations for text

- Bag-of-Words (BOW)
  - Easy, no effort required
  - Variable size, ignores sentential structure

- Hand-crafted features
  - Full control, can use NLP pipeline, class-specific features
  - Over-specific, incomplete, makes use of NLP pipeline

- Learned feature representations [next class]
  - Can learn to contain all relevant information
  - Needs to be learned



Inputs x

Аяны замд тур зогсон тэнгэрийн байдлыг ажиглаад хөдлөх зуур

Београд, 16. јун 2013. године – Председник Владе Републике

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Nestrankarski Urad za vladno odgovornost ZDA je objavil

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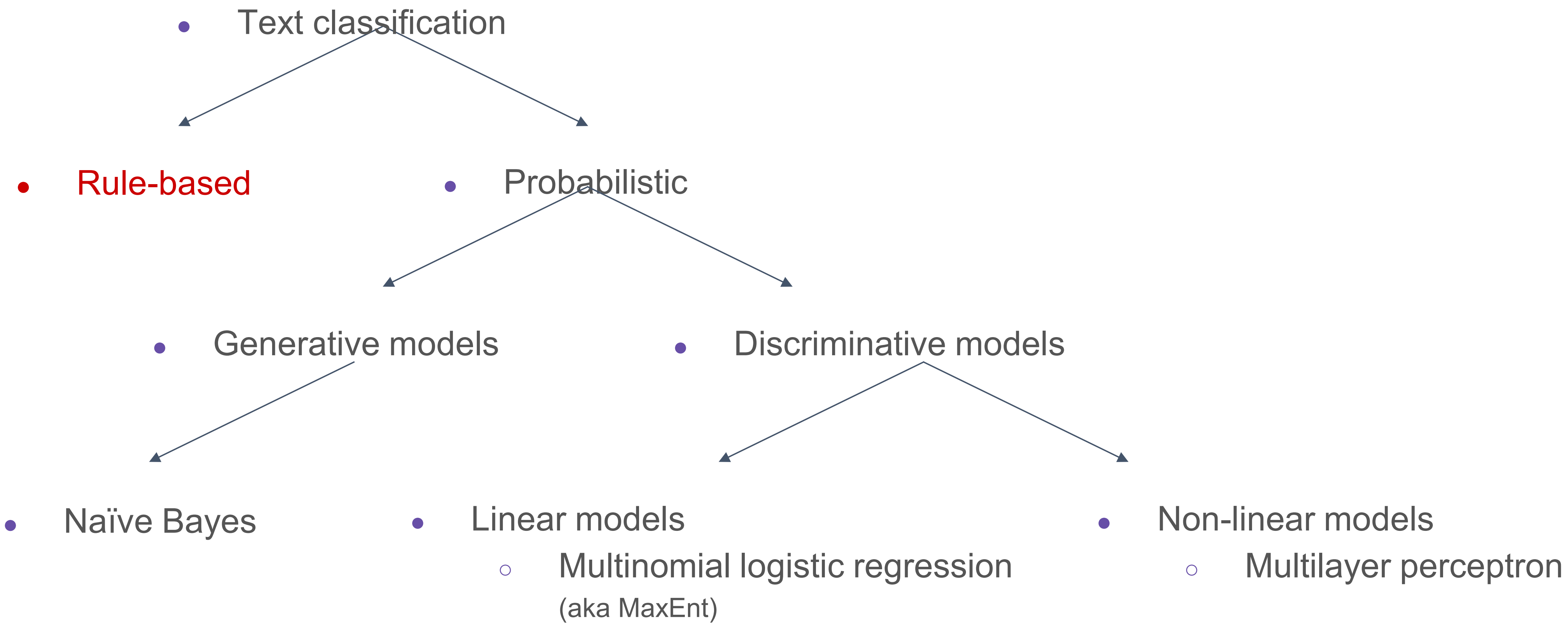
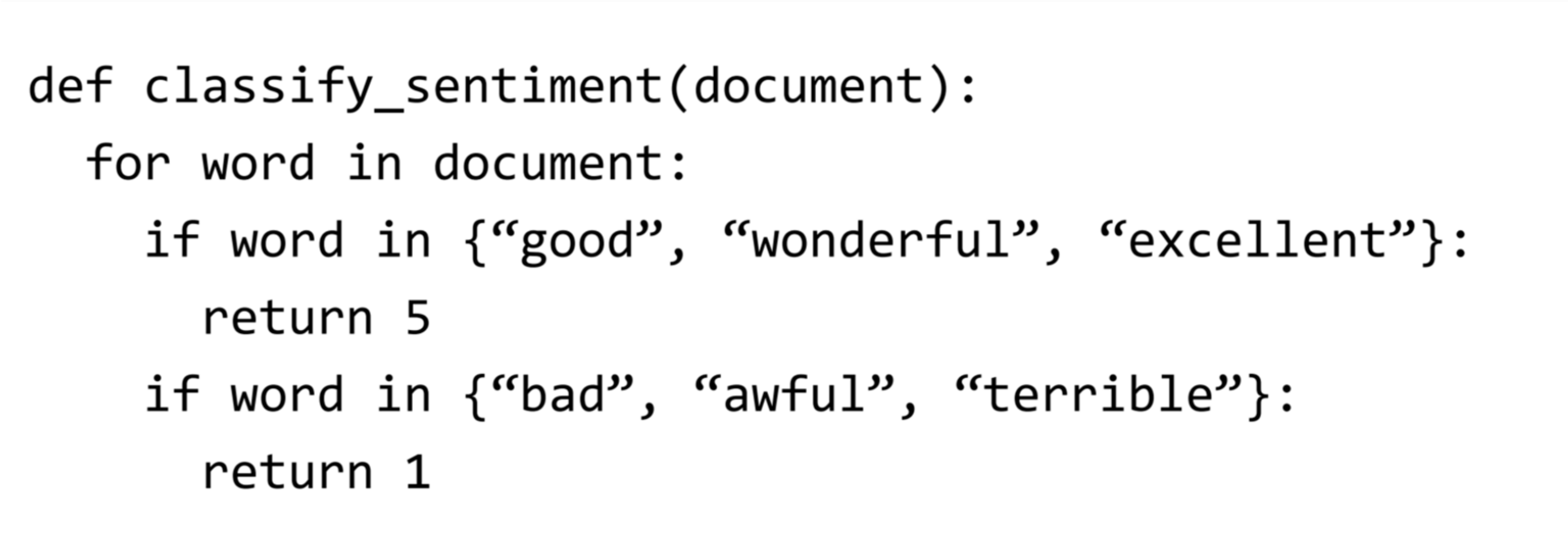
What kinds of strategies might we use to create our function *f*?

Different “models” for classification



- Text classification
- Rule-based
- Probabilistic
- Generative models
- Discriminative models
- Naïve Bayes
- Linear models
    - Multinomial logistic regression (aka MaxEnt)
- Non-linear models
    - Multilayer perceptron

Rule-based classifier



```
def classify_sentiment(document):
  for word in document:
    if word in {“good”, “wonderful”, “excellent”}:
      return 5
    if word in {“bad”, “awful”, “terrible”}:
      return 1
```

Rule-based classification: challenges

Sentiment: Half submarine flick, half ghost story, all in one a criminally neglected film.

Rule-based classification: challenges

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⟶ hard to identify a priori which words are informative (and what information they carry!)

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Sentiment: It’s not life-affirming, it’s vulgar, it’s mean, but I liked it.

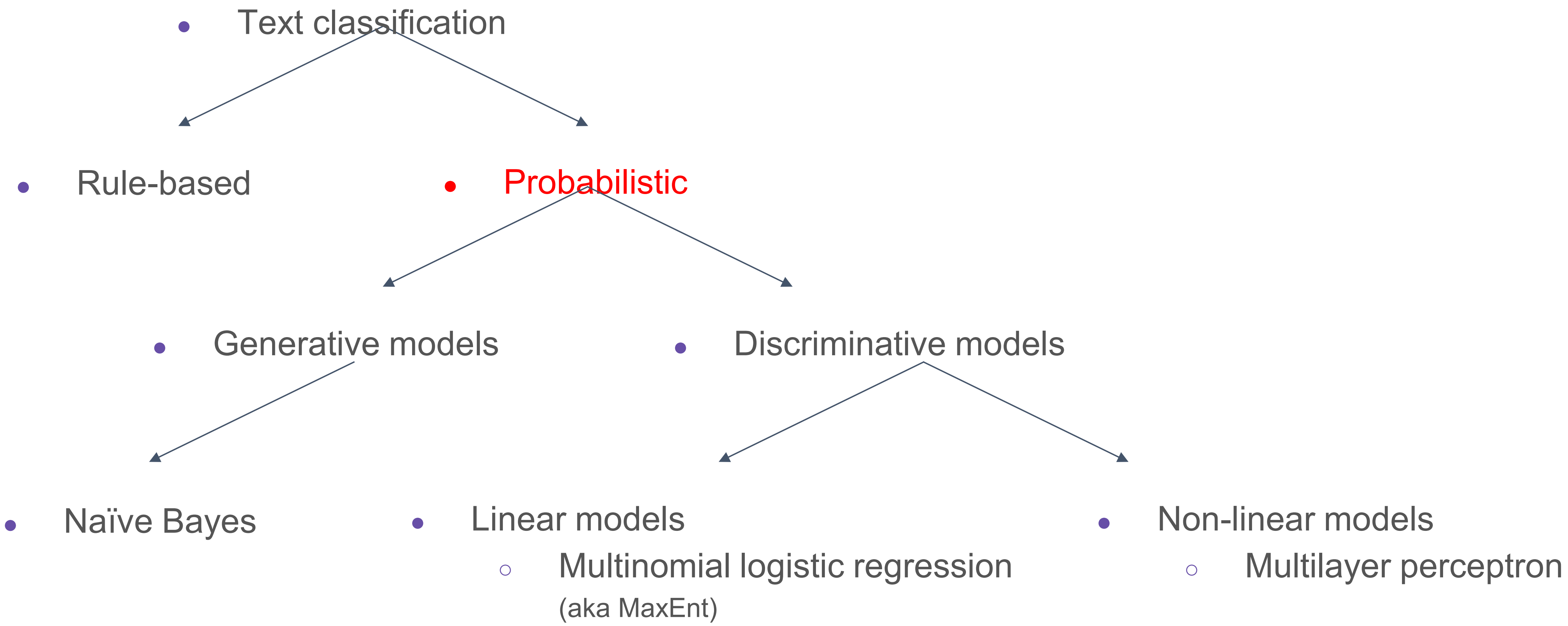
⟶language pragmatics is complex to model at word level, word order (syntax) matters, but hard to encode in rules!

Topic Classification: The team met today to discuss their strategy for the upcoming campaign. ⟶simple features can be misleading!

Rule-based classification

But don’t forget: if you don’t have access to data, speaker intuition and a bit of coding get you pretty far!

We’ll consider alternative models for classification



- Text classification

- Rule-based
- Probabilistic

- Generative models
- Discriminative models

- Naïve Bayes
- Linear models
  - Multinomial logistic regression (aka MaxEnt)
- Non-linear models
  - Multilayer perceptron

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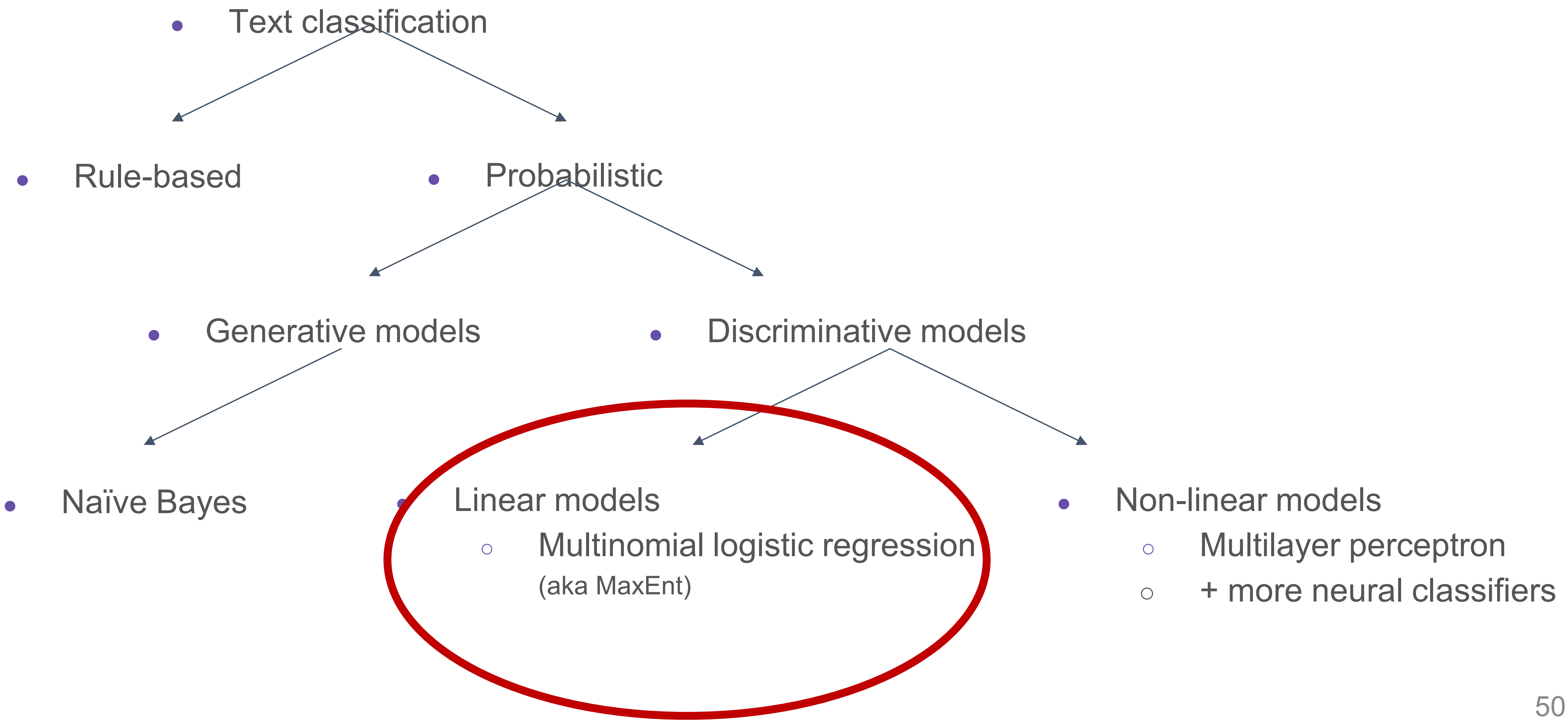

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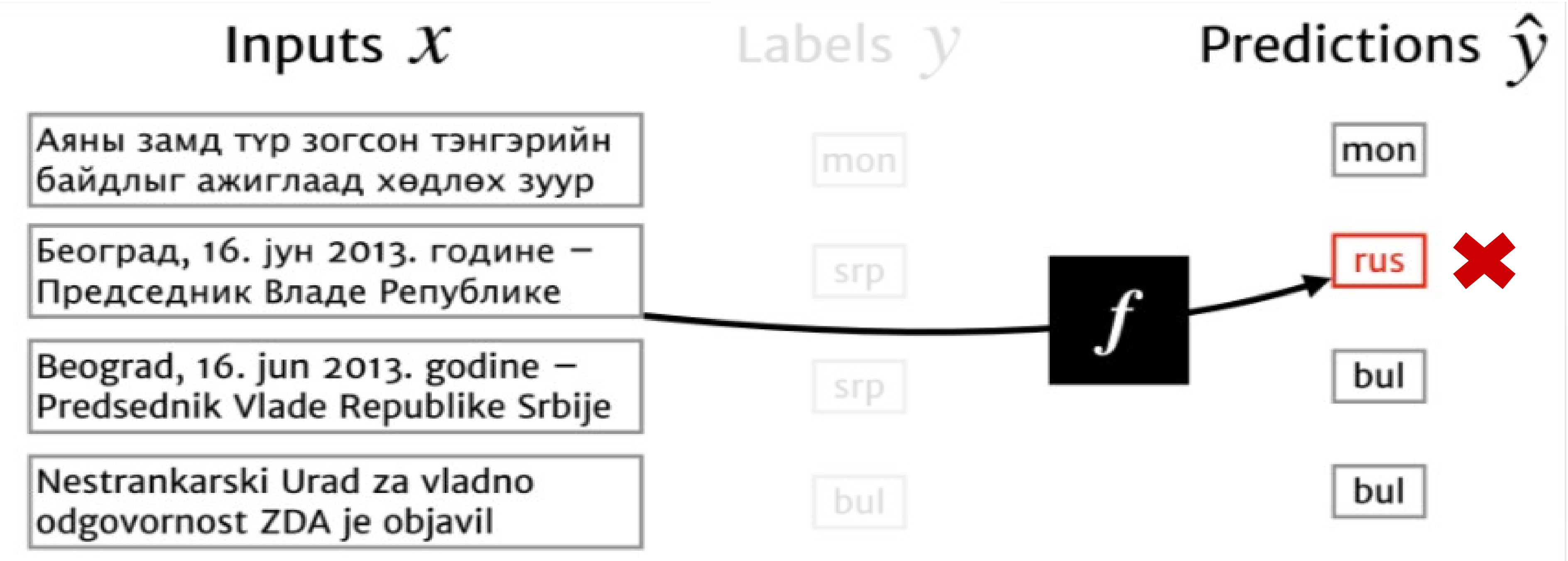
  - Multilayer perceptron
  - + more neural classifiers

Learning-based classification

Goal: create a function f that makes a prediction $\hat{y}$ given an input x



Inputs x

Labels y

Predictions $\hat{y}$

Аяны замд түр зогсон тэнгэрийн байдлыг ажиглаад хөдлөх зуур

Београд, 16. јун 2013. године – Председник Владе Републике

Beograd, 16. jun 2013. godine – Predsednik Vlade Republike Srbije

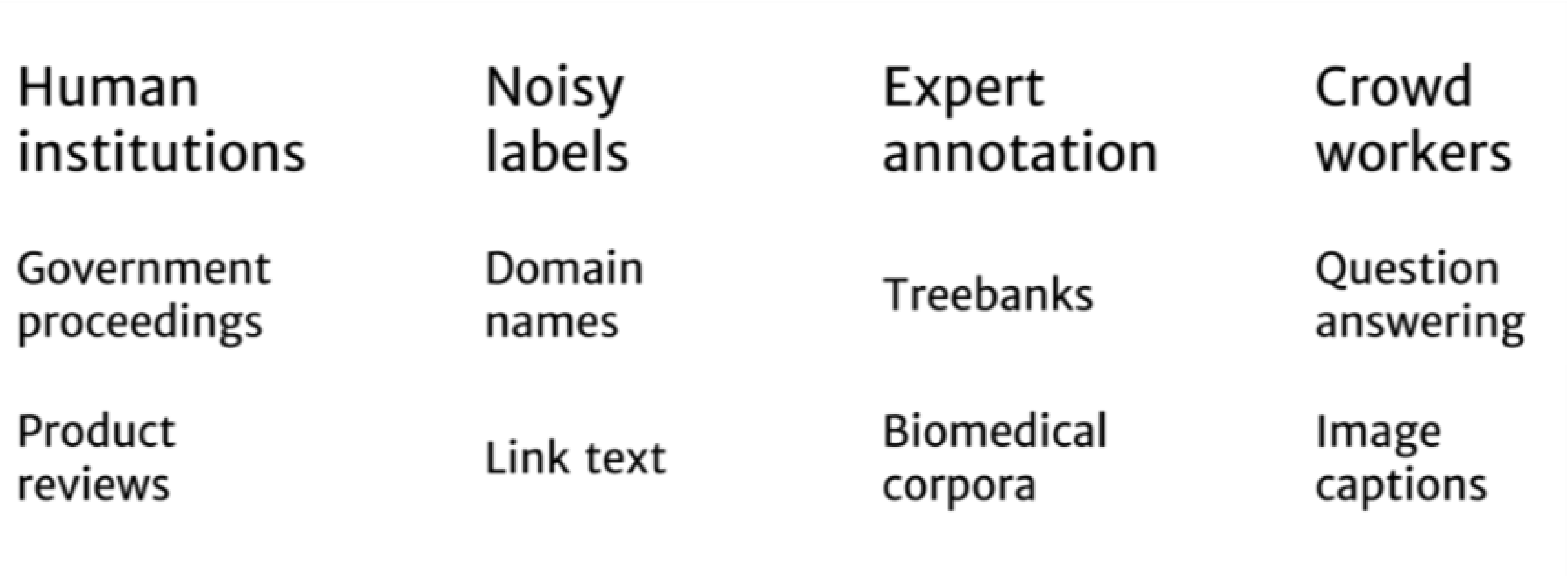
Nestrankarski Urad za vladno odgovornost ZDA je objavil

pick the function f that does “best” on training data

Classification: learning from data

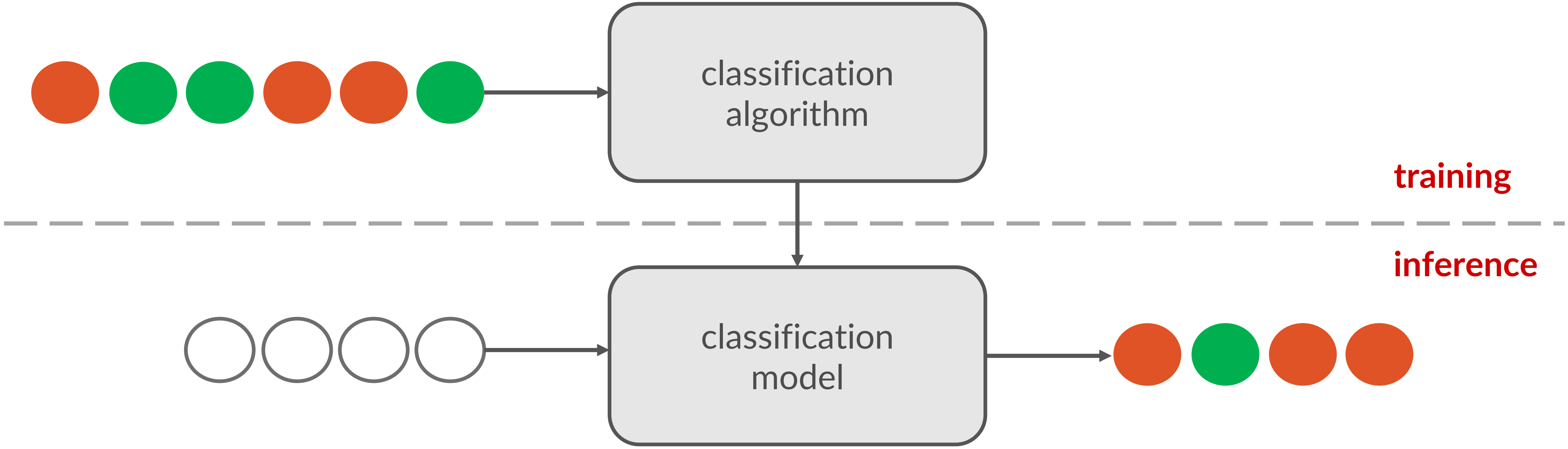
- Supervised
  - labeled examples
    - Binary (true, false)
    - Multi-class classification (politics, sports, gossip)
    - Multi-label classification (#party #FRIDAY #fail)
- Unsupervised
  - no labeled examples
- Semi-supervised
  - labeled examples + non-labeled examples
- Weakly supervised
  - heuristically-labeled examples

Where do datasets come from?

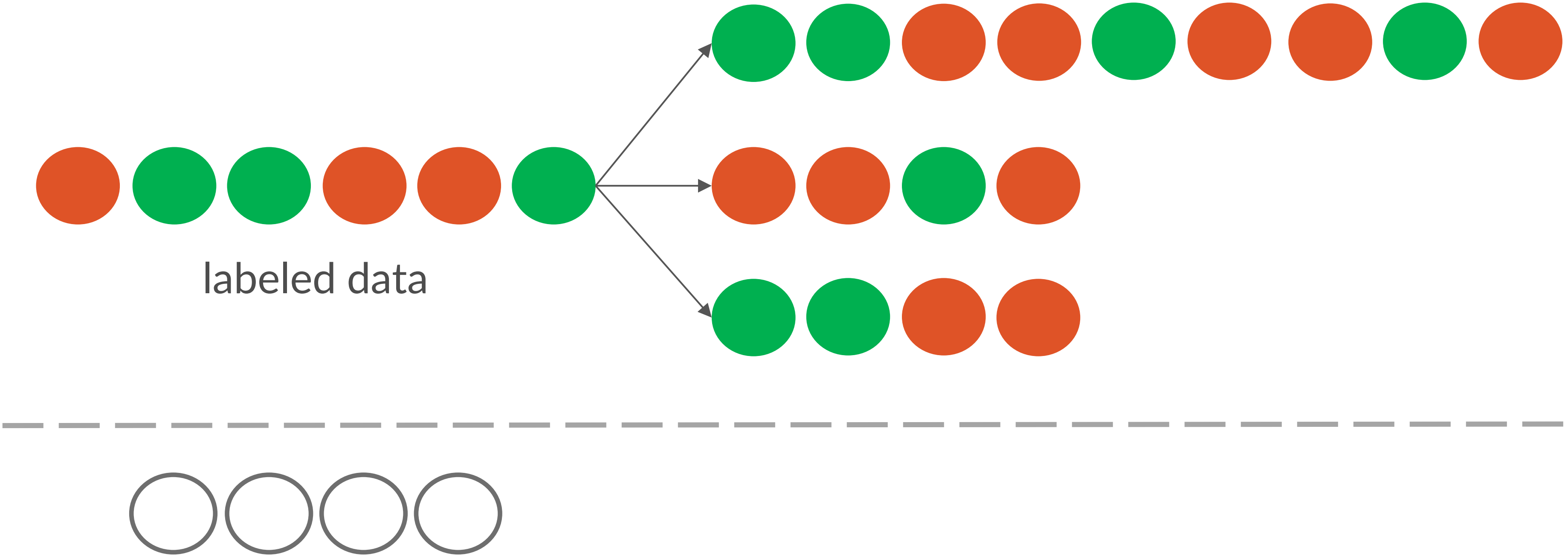


Human institutions	Noisy labels	Expert annotation	Crowd workers
Government proceedings	Domain names	Treebanks	Question answering
Product reviews	Link text	Biomedical corpora	Image captions

Supervised classification



Training, validation, and test sets



training set

training

validation set

labeled data

test set

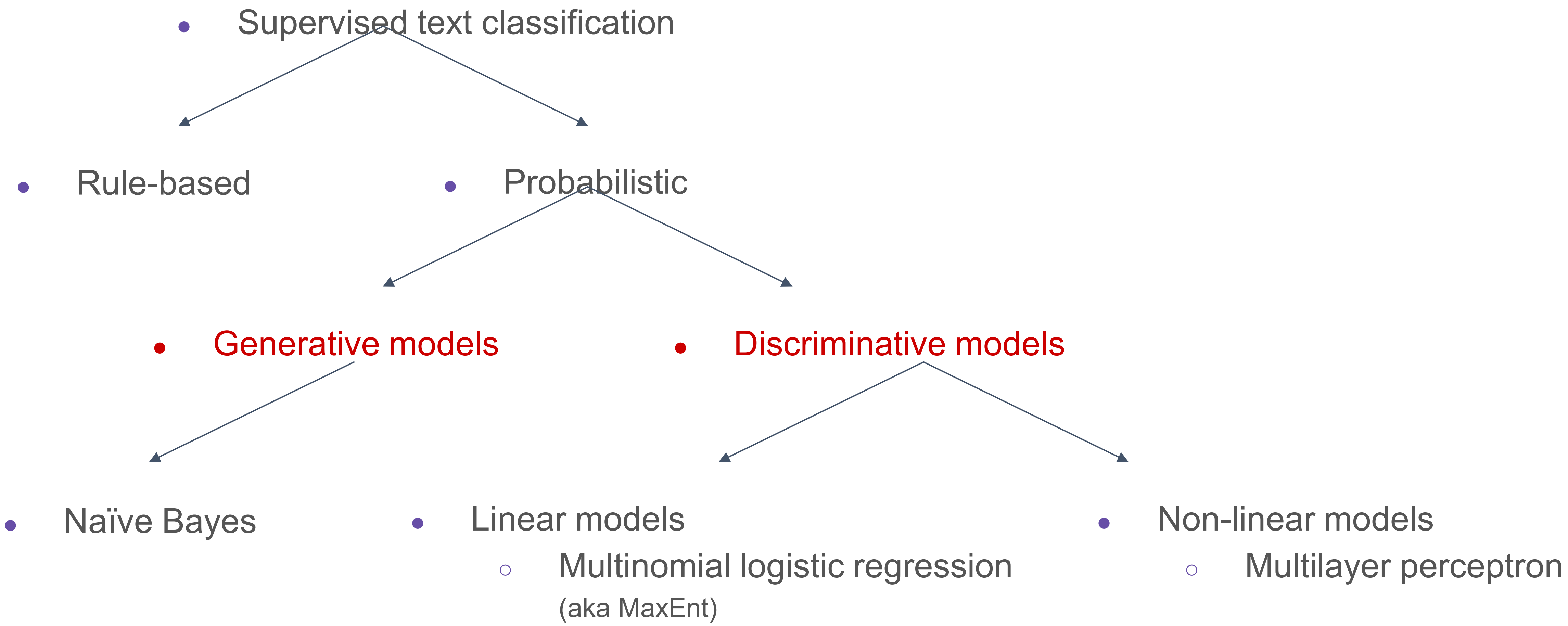
inference

unlabeled data

Supervised classification: formal setting

- Learn a classification model from labeled data on
    - properties ("features") and their importance ("weights")
- X: set of attributes or features $\{x_1, x_2, \ldots, x_n\}$
    - e.g. fruit measurements, or word counts extracted from an input documents
- y: a "class" label from the label set $Y = \{y_1, y_2, \ldots, y_k\}$
    - e.g., fruit type, or spam/not spam, positive/negative/neutral

We’ll consider alternative models for classification



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Generative and discriminative models

- Generative model: a model that calculates the probability of the input data itself

$$P(X, Y)$$

joint

- Discriminative model: a model that calculates the probability of a latent trait given the data

$$P(Y \mid X)$$

conditional

Generative and discriminative models

imagenet

imagenet

Generative model

- Build a model of what's in a cat image
  - Knows about whiskers, ears, eyes
  - Assigns a probability to any image:
  - how cat-y is this image?
- Also build a model for dog images

imagenet

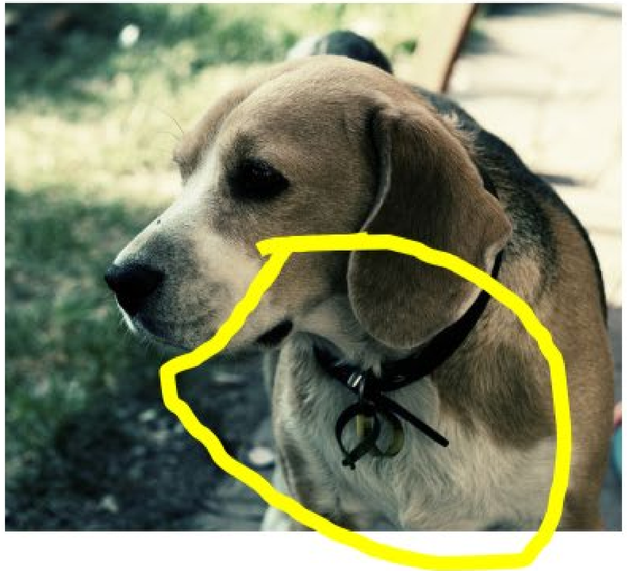
imagenet

Now given a new image:

Run both models and see which one fits better

Discriminative model

Just try to distinguish dogs from cats



Oh look, dogs have collars! Let's ignore everything else

Generative and discriminative models

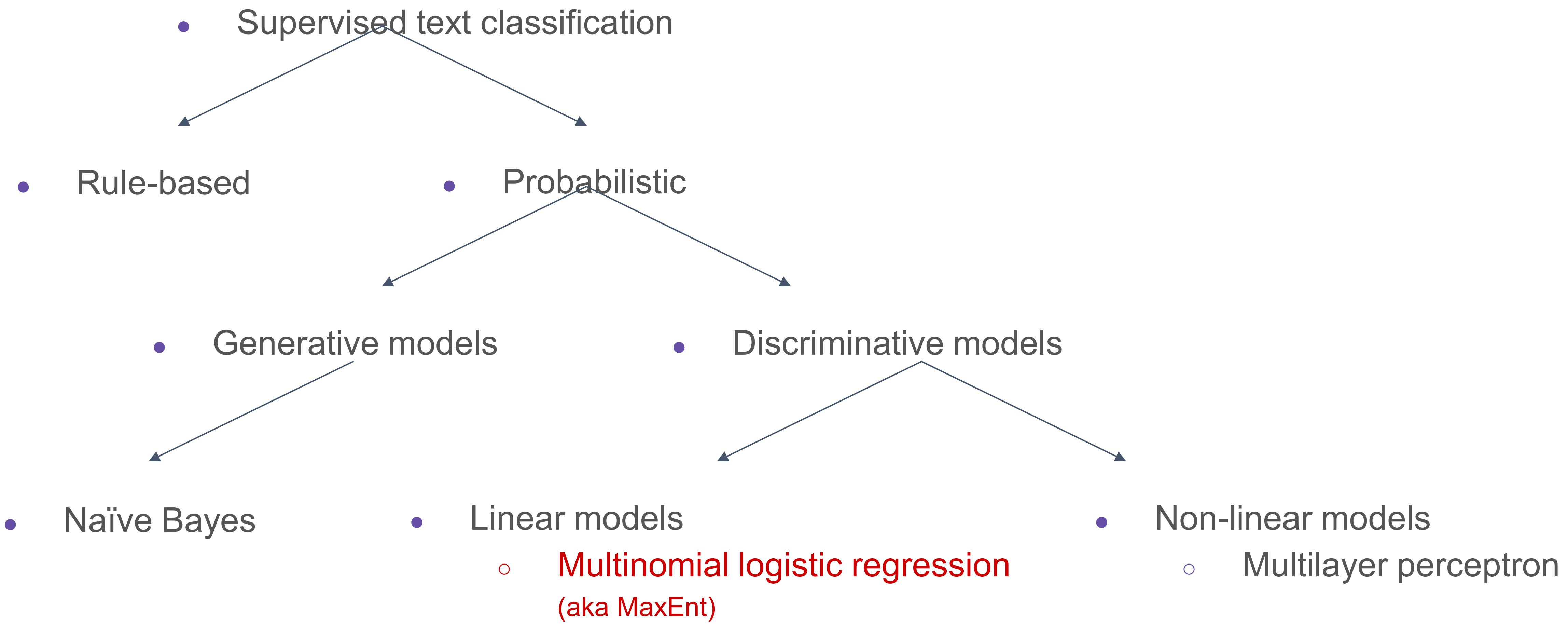
- Generative text classification: Learn a model of the joint P(X, y), and find

$$\hat{y} = \underset{\tilde{y}}{\operatorname{argmax}}\ P(X, \tilde{y})$$

- Discriminative text classification: Learn a model of the conditional P(y | X), and find

$$\hat{y} = \underset{\tilde{y}}{\operatorname{argmax}}\ P(\tilde{y}|X)$$

Logistic regression



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Logistic regression classifier

- Important analytic tool in natural and social sciences
- Baseline supervised machine learning tool for classification
- Is also the foundation of neural networks

Binary classification in logistic regression

- Given a series of input/output pairs:
  - $(x^{(i)}, y^{(i)})$

- For each observation $x^{(i)}$
  - We represent $x^{(i)}$ by a feature vector $\{x_1, x_2, \ldots, x_n\}$
  - We compute an output: a predicted class $\hat{y}^{(i)} \in \{0,1\}$

Features in logistic regression

- For feature $x_i \in \{x_1, x_2, \ldots, x_n\}$, weight $w_i \in \{w_1, w_2, \ldots, w_n\}$ tells us how important is x_i
  - x_i = "review contains 'awesome'": w_i = +10
  - x_j = "review contains horrible": w_j = -10
  - x_k = "review contains 'mediocre'": w_k = -2

How to do classification

- For each feature x_i, weight w_i tells us importance of x_i
  - (Plus we'll have a bias b)
  - We'll sum up all the weighted features and the bias

$$
\begin{aligned}
z &= \left(\sum_{i=1}^{n} w_i x_i\right) + b \\
z &= w \cdot x + b
\end{aligned}
$$

If this sum is high, we say $y=1$; if low, then $y=0$

Formalizing “sum is high”

- We’d like a principled classifier that gives us a probability

- We want a model that can tell us:
  - $p(y=1|x;\theta)$
  - $p(y=0|x;\theta)$

The problem: z isn't a probability, it's just a number!

- z ranges from $-\infty$ to ∞

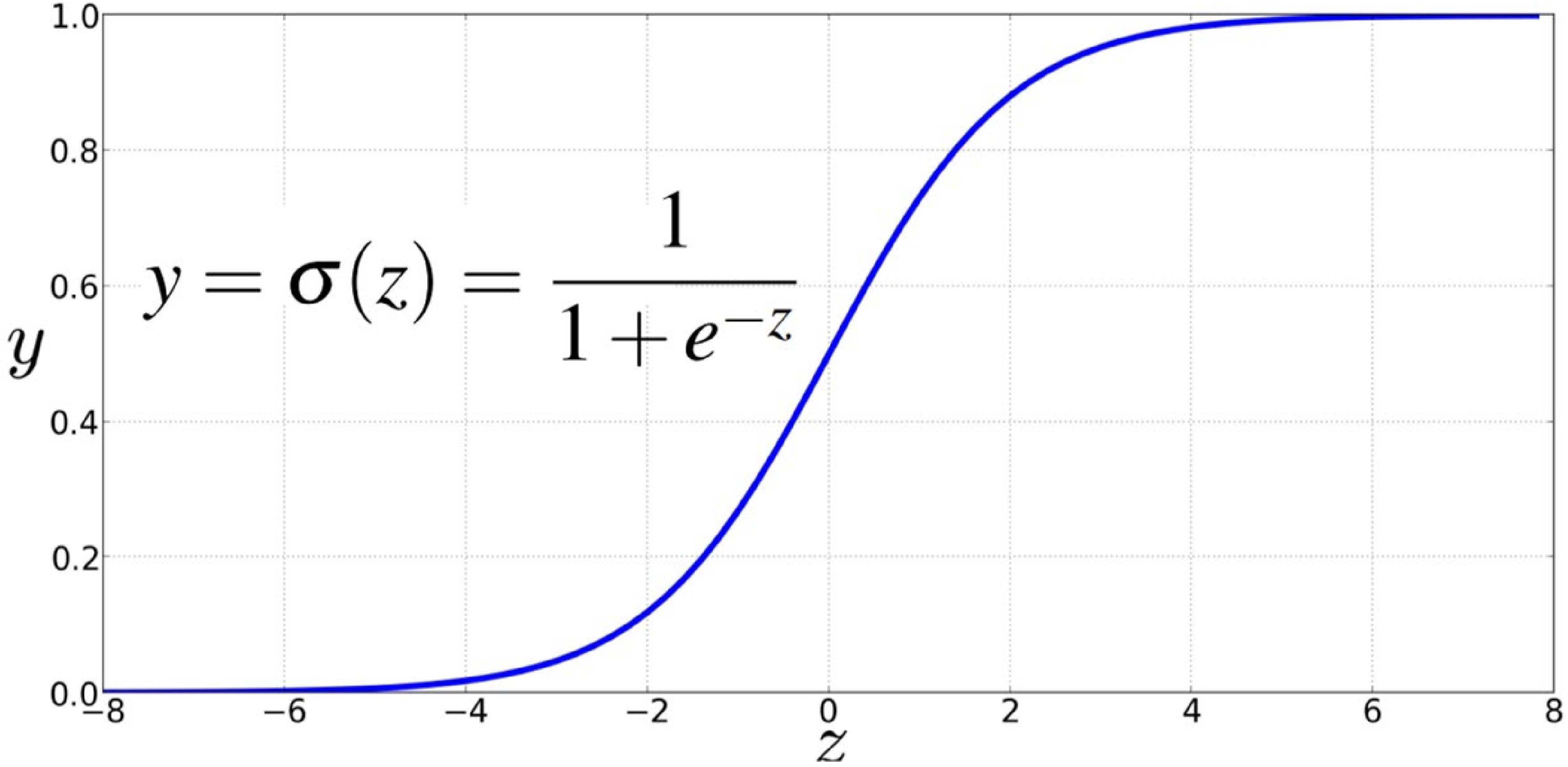
$$z = w \cdot x + b$$

- Solution: use a function of z that goes from 0 to 1

“sigmoid” or
“logistic” function

$$y = \sigma(z) = \frac{1}{1+e^{-z}} = \frac{1}{1+\exp(-z)}$$

The very useful sigmoid or logistic function



$$y = \sigma(z) = \frac{1}{1+e^{-z}}$$

Idea of logistic regression

- We’ll compute w·x+b
- And then we’ll pass it through the sigmoid function:

$$\sigma(w \cdot x+b)$$

- And we'll just treat it as a probability

Making probabilities with sigmoids

$$
\begin{aligned}
P(y=1) &= \sigma(w \cdot x + b) \\
&= \frac{1}{1+\exp\left(-(w \cdot x + b)\right)}
\end{aligned}
$$

Making probabilities with sigmoids

$$\begin{aligned} P(y=1) &= \sigma(w\cdot x+b) \\ &= \frac{1}{1+\exp\left(-(w\cdot x+b)\right)} \end{aligned}$$

$$\begin{aligned} P(y=0) &= 1-\sigma(w\cdot x+b) \\ &= 1-\frac{1}{1+\exp\left(-(w\cdot x+b)\right)} \\ &= \frac{\exp\left(-(w\cdot x+b)\right)}{1+\exp\left(-(w\cdot x+b)\right)} \end{aligned}$$

By the way:

$$\begin{aligned} P(y=0) &= 1-\sigma(w\cdot x+b) \\ &= 1-\frac{1}{1+\exp\left(-(w\cdot x+b)\right)} \\ &= \frac{\exp\left(-(w\cdot x+b)\right)}{1+\exp\left(-(w\cdot x+b)\right)} \end{aligned}$$

$$= \sigma(-(w\cdot x+b))$$

Because

$$\underline{1-\sigma(x)=\sigma(-x)}$$

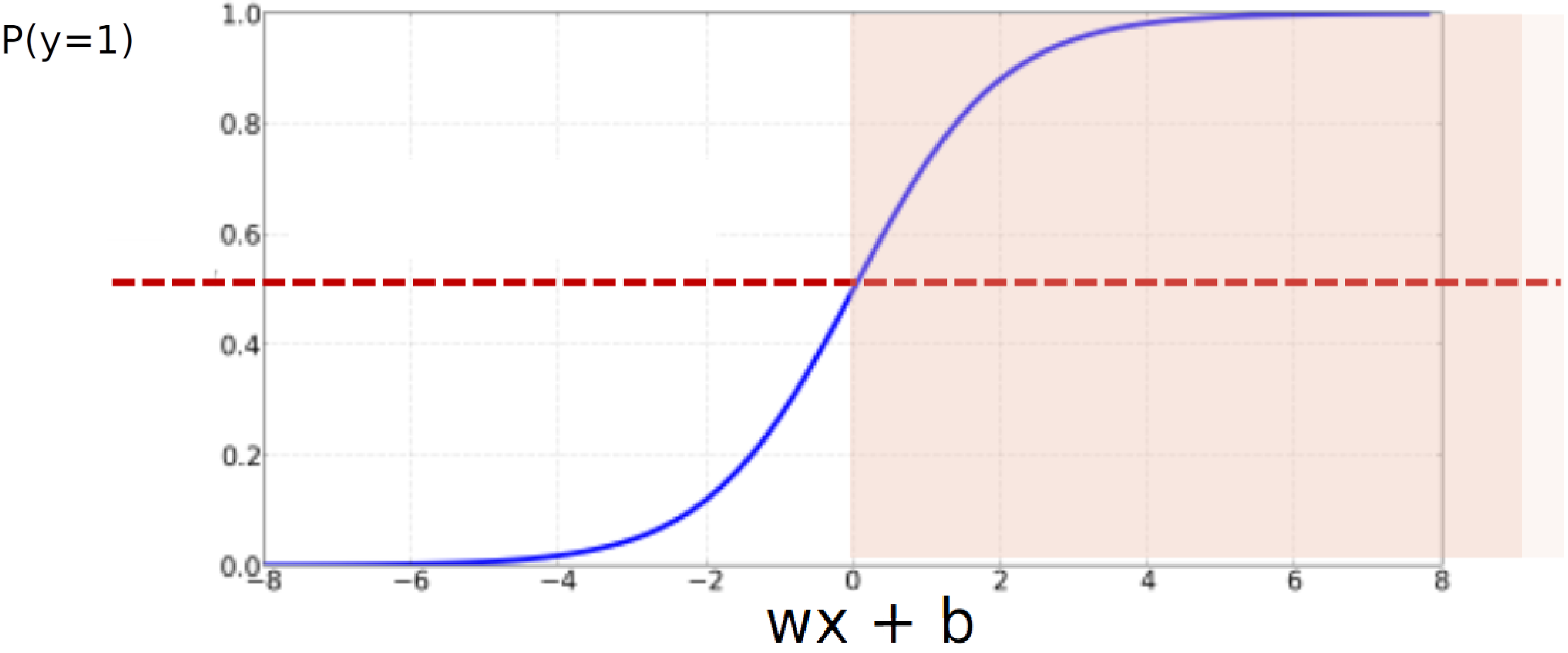
Turning a probability into a classifier

- 0.5 here is called the **decision boundary**

$$\hat{y} = \begin{cases} 1 & \text{if } P(y=1|x) > 0.5 \\ 0 & \text{otherwise} \end{cases}$$

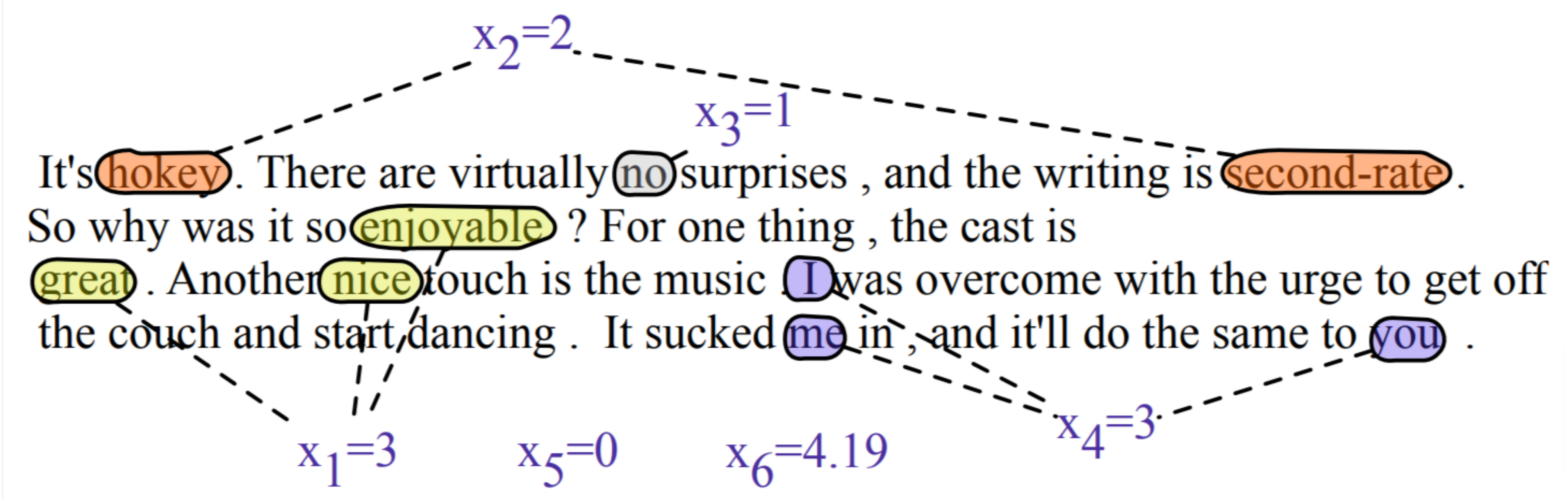
The probabilistic classifier

$$P(y=1) = \sigma(w \cdot x + b)$$
$$= \frac{1}{1+\exp(-(w \cdot x + b))}$$



Sentiment example: does y=1 or y=0?

It's hokey . There are virtually no surprises , and the writing is second-rate . So why was it so enjoyable ? For one thing , the cast is great . Another nice touch is the music . I was overcome with the urge to get off the couch and start dancing . It sucked me in , and it'll do the same to you .



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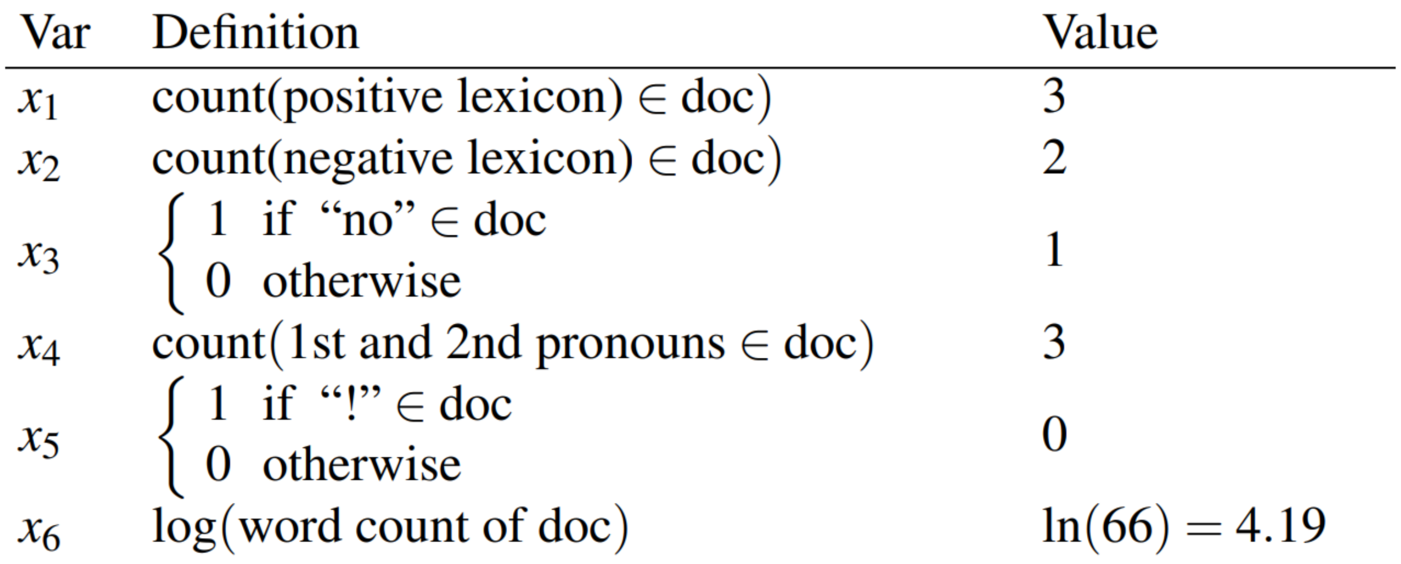
Var	Definition	Value
x_1	count(positive lexicon) $\in$ doc)	3
x_2	count(negative lexicon) $\in$ doc)	2
x_3	$\begin{cases} 1 & \text{if “no”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1
x_4	count(1st and 2nd pronouns $\in$ doc)	3
x_5	$\begin{cases} 1 & \text{if “!”} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	0
x_6	log(word count of doc)	$\ln(66) = 4.19$

Classifying sentiment for input x

Suppose

w = [2.5, -5.0, -1.2, 0.5, 2.0, 0.7]
b = 0.1



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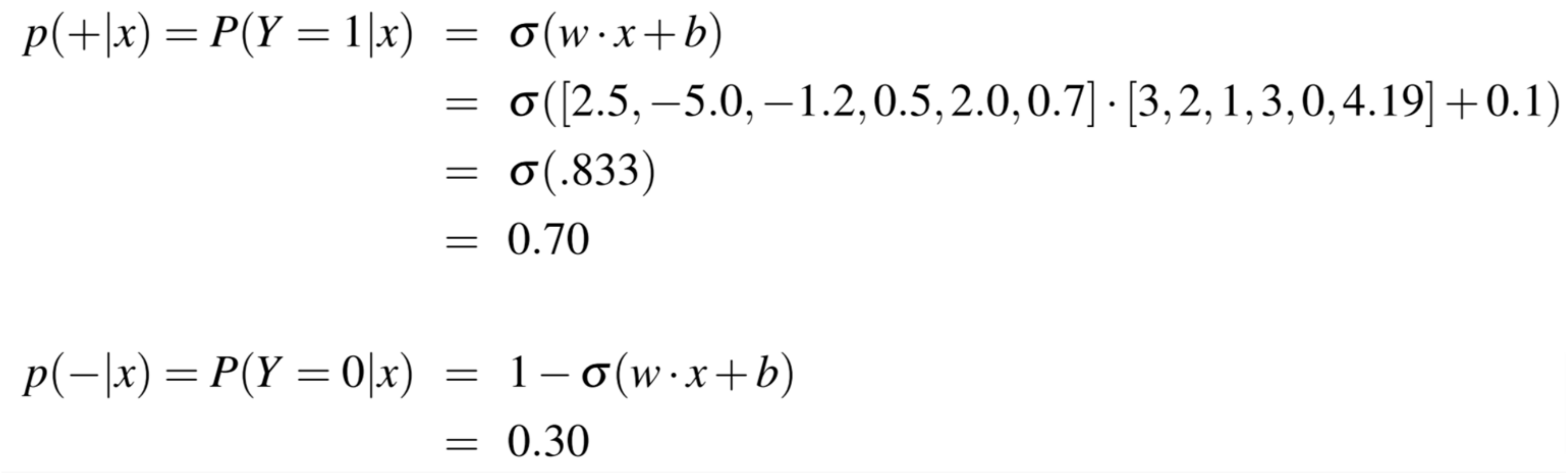

$$\begin{aligned}
p(+|x) = P(Y=1|x) &= \sigma(w \cdot x + b) \\
&= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.19] + 0.1) \\
&= \sigma(.833) \\
&= 0.70
\end{aligned}$$

$$\begin{aligned}
p(-|x) = P(Y=0|x) &= 1 - \sigma(w \cdot x + b) \\
&= 0.30
\end{aligned}$$

Wait, where did the W's come from?

- Supervised classification:
  - At training time, we know the correct label y (either 0 or 1) for each x.
  - But what the system produces at inference time is an estimate $\hat{y}$

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- Supervised classification:
    - At training time, we know the correct label y (either 0 or 1) for each x.
    - But what the system produces at inference time is an estimate $\hat{y}$

- We want to set w and b to minimize the **distance** between our estimate $\hat{y}^{(i)}$ and the true $y^{(i)}$
    - We need a distance estimator: a **loss function** or an objective function
    - We need an **optimization algorithm** to update w and b to minimize the loss

Learning components in LR

A loss function:

- **cross-entropy loss**

An optimization algorithm:

- **stochastic gradient descent**

Loss function: the distance between ŷ and y

We want to know how far is the classifier output ŷ $= \sigma(w \cdot x + b)$

from the true output: y [= either 0 or 1]

We'll call this difference: L(ŷ,y) = how much ŷ differs from the true y

Training Objective: Maximize the Likelihood of the Training data.

We choose the parameters w,b that maximize

- the probability (aka likelihood)
- of the true y labels in the training data
- given the observations x

Deriving the objective for a single observation x

Goal: maximize probability of the correct label under the model

The predicted probability for class 1 is ŷ.

If the correct label is 1, then the likelihood is ŷ. Else it is 1-ŷ

We can express the probability p(y|x) from our classifier (the thing we want to maximize) as

$$p(y|x) = \hat{y}^{y}\,(1-\hat{y})^{1-y}$$

Deriving the objective for a single observation x

Goal: maximize probability of the correct label $p(y|x)$

$$p(y|x) = \hat{y}^{y} (1 - \hat{y})^{1-y}$$

Noting:

if $y=1$, this simplifies to $\hat{y}$
if $y=0$, this simplifies to $1 - \hat{y}$

Deriving the objective for a single observation x

Goal: maximize probability of the correct label $p(y|x)$

Maximize: $p(y|x) \;=\; \hat{y}^{y}\,(1-\hat{y})^{1-y}$

Deriving the objective for a single observation x

Goal: maximize probability of the correct label p(y|x)

Maximize: $p(y|x) = \hat{y}^y (1-\hat{y})^{1-y}$

Now take the log of both sides (mathematically handy)

Maximize:

$$\begin{aligned} \log p(y|x) &= \log \left[\hat{y}^y (1-\hat{y})^{1-y}\right] \\ &= y \log \hat{y} + (1-y) \log(1-\hat{y}) \end{aligned}$$

Deriving cross-entropy loss for a single observation x

Goal: maximize probability of the correct label p(y|x)

Maximize: $p(y|x) \;=\; \hat{y}^{y}\,(1-\hat{y})^{1-y}$

Now take the log of both sides (mathematically handy)

Maximize:

$$\begin{aligned} \log p(y|x) &= \log\left[\hat{y}^{y}\,(1-\hat{y})^{1-y}\right] \\ &= y\log\hat{y} + (1-y)\log(1-\hat{y}) \end{aligned}$$

Whatever values maximize log p(y|x) will also maximize p(y|x)

Deriving the objective for a single observation x

Goal: maximize probability of the correct label p(y|x)

Maximize:
$$\begin{aligned}\log p(y|x) &= \log\left[\hat{y}^{y}\,(1-\hat{y})^{1-y}\right]\\ &= y\log\hat{y}+(1-y)\log(1-\hat{y})\end{aligned}$$

Now flip sign to turn this into a loss: something to minimize

Deriving the objective for a single observation x

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Now flip sign to turn this into a loss: something to minimize

Minimize:

$$L_{\mathrm{CE}}(\hat{y},y) = -\log p(y|x) \;=\; -[y\log\hat{y}+(1-y)\log(1-\hat{y})]$$

Deriving the objective for a single observation x

Goal: maximize probability of the correct label p(y|x)

Maximize:
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Now flip sign to turn this into a **cross-entropy loss**: something to minimize

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$$L_{\mathrm{CE}}(\hat{y},y) = -\log p(y|x) \;=\; -\left[y\log\hat{y}+(1-y)\log(1-\hat{y})\right]$$

Deriving cross-entropy loss for a single observation x

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Minimiz

$$L_{\mathrm{CE}}(\hat{y},y)=-\log p(y|x) \;=\; -[y\log\hat{y}+(1-y)\log(1-\hat{y})]$$

Or, plug in definition of $\hat{y} = \sigma(w{\cdot}x{+}b)$

$$\boxed{L_{\mathrm{CE}}(\hat{y},y) \;=\; -[y\log\sigma(\mathbf{w}\cdot\mathbf{x}+b)+(1-y)\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))]}$$

Our goal: minimize the loss

Let's make explicit that the loss function is parameterized by weights θ=(w,b)

- And we'll represent ŷ as f(x; θ) to make the dependence on θ more obvious

We want the weights that minimize the loss, averaged over all examples:

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \frac{1}{m} \sum_{i=1}^{m} L_{\mathrm{CE}}(f(x^{(i)}; \theta), y^{(i)})$$

$$L_{\mathrm{CE}}(\hat{y}, y)$$

Let's see if this works for our sentiment example

We want loss to be:

- smaller if the model estimate $\hat{y}$ is close to correct
- bigger if model is confused

Let's first suppose the true label of this is $y=1$ (positive)

It's hokey . There are virtually no surprises , and the writing is second-rate . So why was it so enjoyable ? For one thing , the cast is great . Another nice touch is the music . I was overcome with the urge to get off the couch and start dancing . It sucked me in , and it'll do the same to you .

Let's see if this works for our sentiment example

True value is y=1 (positive). How well is our model doing?

$$\begin{aligned} p(+|x) = P(Y=1|x) &= \sigma(w \cdot x + b) \\ &= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.19] + 0.1) \\ &= \sigma(.833) \\ &= 0.70 \end{aligned}$$

Pretty well!

Let's see if this works for our sentiment example

True value is y=1 (positive). How well is our model doing?

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&= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.19] + 0.1) \\
&= \sigma(.833) \\
&= 0.70
\end{aligned}
$$

Pretty well! What's the loss?

$$
\begin{aligned}
L_{\mathrm{CE}}(\hat{y}, y) = & -[y \log \sigma(\mathbf{w} \cdot \mathbf{x} + b) + (1-y) \log (1 - \sigma(\mathbf{w} \cdot \mathbf{x} + b))] \\
= & -[\log \sigma(\mathbf{w} \cdot \mathbf{x} + b)] \\
= & -\log(.70) \\
= & .36
\end{aligned}
$$

Let's see if this works for our sentiment example

Suppose the true value instead was $y=0$ (negative).

$$\begin{aligned} p(-|x) = P(Y=0|x) &= 1-\sigma(w \cdot x + b) \\ &= 0.30 \end{aligned}$$

Let's see if this works for our sentiment example

Suppose the true value instead was y=0 (negative).

$$
\begin{aligned}
p(-|x) = P(Y=0|x) &= 1-\sigma(w\cdot x+b) \\
&= 0.30
\end{aligned}
$$

What's the loss?

$$
\begin{aligned}
L_{\mathrm{CE}}(\hat{y},y) &= -[y\log\sigma(\mathbf{w}\cdot\mathbf{x}+b)+(1-y)\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))] \\
&= -[\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))] \\
&= -\log(.30) \\
&= 1.2
\end{aligned}
$$

Let's see if this works for our sentiment example

The loss when the model was right (if true y=1)

$$\begin{aligned} L_{\mathrm{CE}}(\hat{y},y) &= -[y\log\sigma(\mathbf{w}\cdot\mathbf{x}+b)+(1-y)\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))] \\ &= -[\log\sigma(\mathbf{w}\cdot\mathbf{x}+b)] \\ &= -\log(.70) \\ &= .36 \end{aligned}$$

The loss when the model was wrong (if true y=0)

$$\begin{aligned} L_{\mathrm{CE}}(\hat{y},y) &= -[y\log\sigma(\mathbf{w}\cdot\mathbf{x}+b)+(1-y)\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))] \\ &= -[\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))] \\ &= -\log(.30) \\ &= 1.2 \end{aligned}$$

Sure enough, loss was bigger when model was wrong!

Learning components

A loss function:

- **cross-entropy loss**

An optimization algorithm:

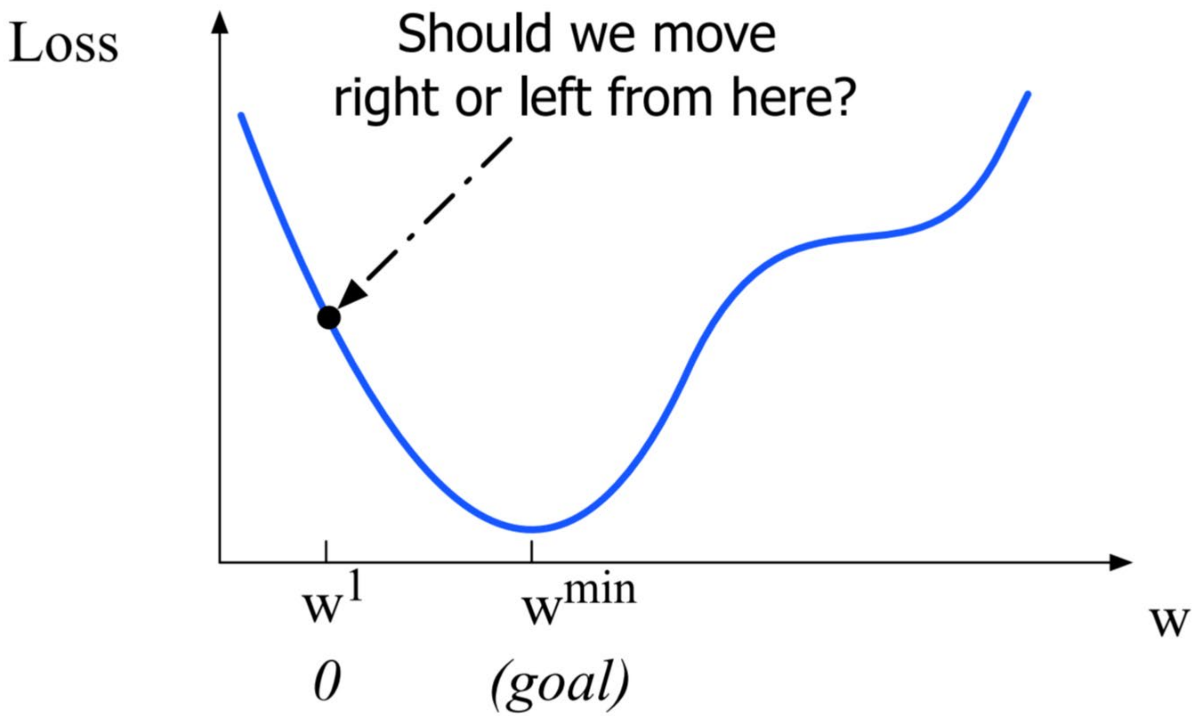
- **stochastic gradient descent**

Gradient Descent

- Gradient Descent algorithm
  - is used to optimize the weights for a machine learning model

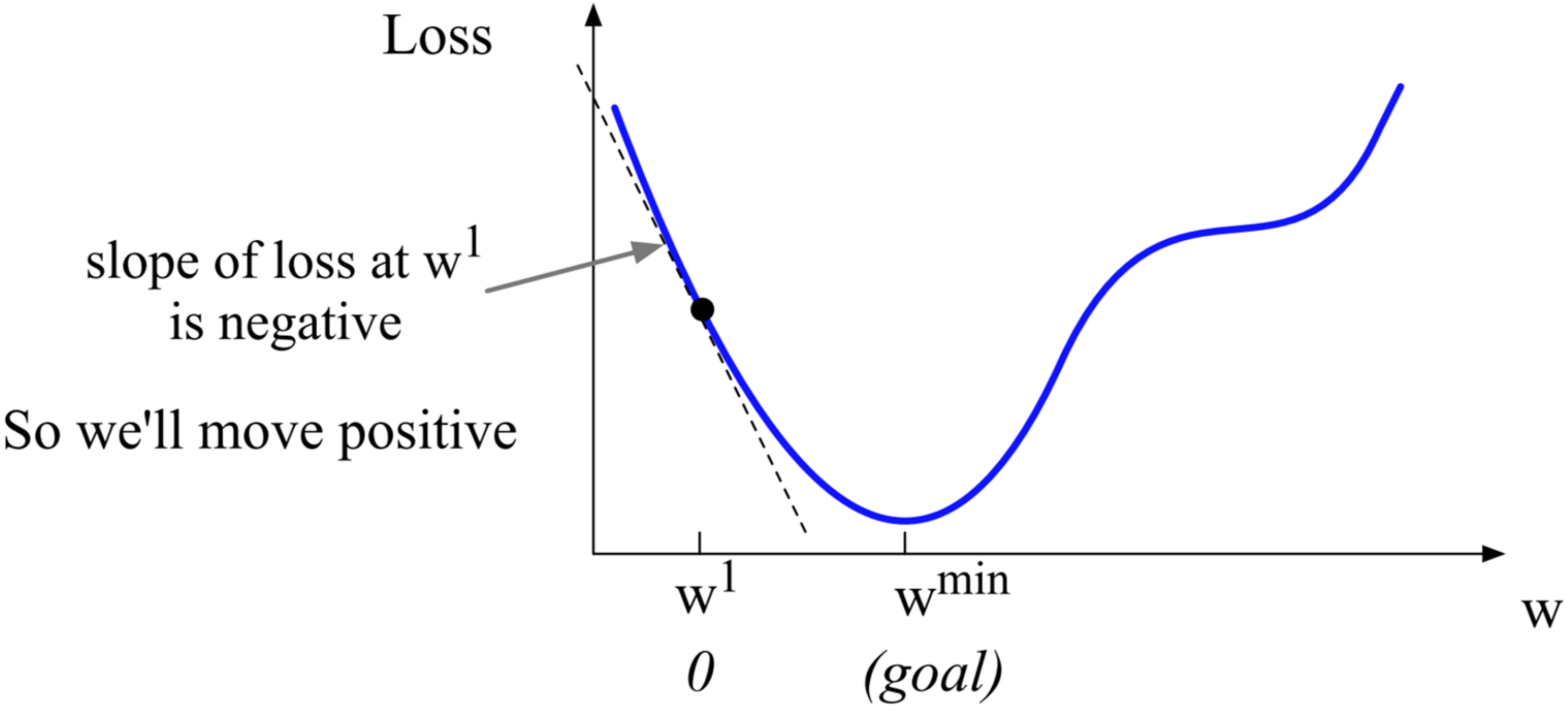
Let's first visualize for a single scalar w

Q: Given current w, should we make it bigger or smaller?
A: Move w in the reverse direction from the slope of the function



Let's first visualize for a single scalar w

Q: Given current w, should we make it bigger or smaller?
A: Move w in the reverse direction from the slope of the function



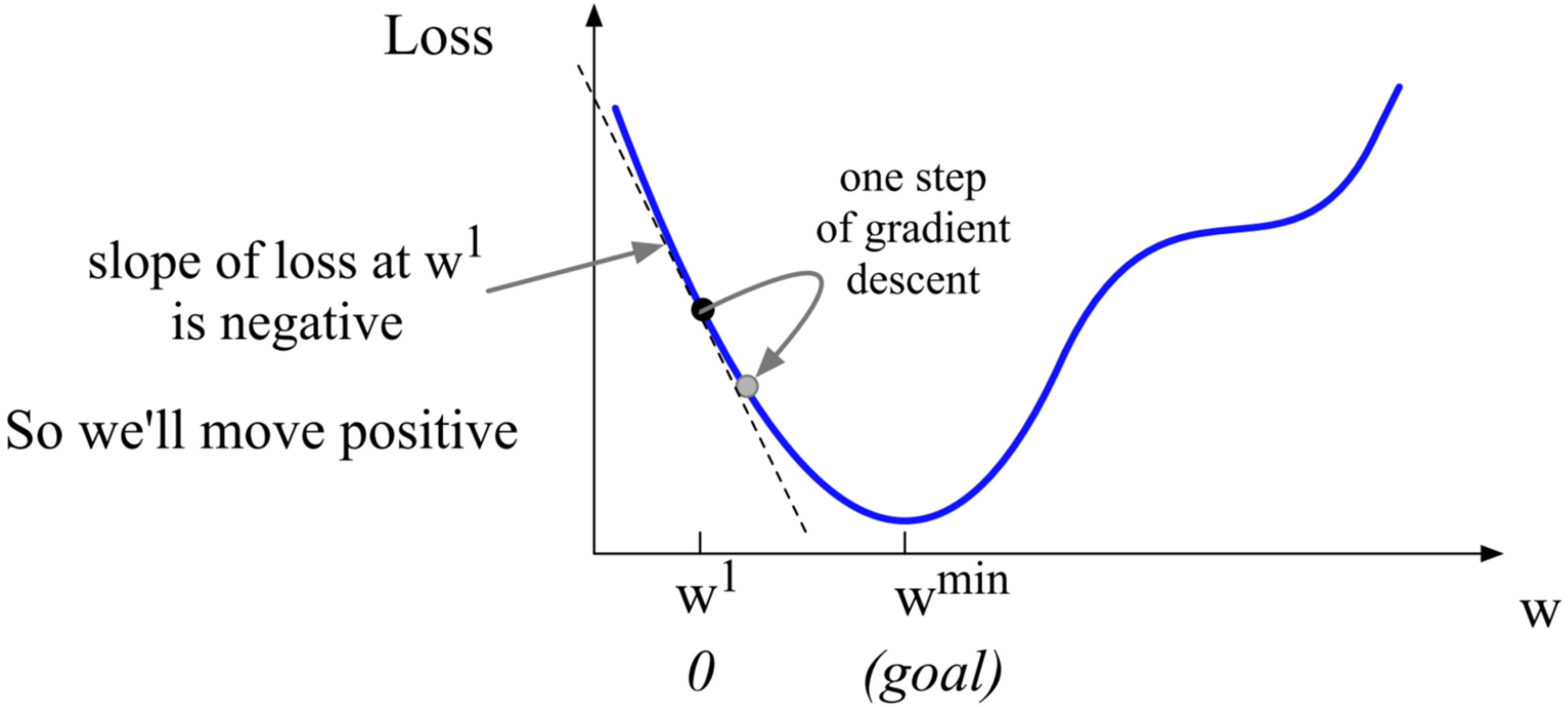
slope of loss at w^1
is negative

So we'll move positive

Let's first visualize for a single scalar w

Q: Given current w, should we make it bigger or smaller?

A: Move w in the reverse direction from the slope of the function



Our goal: minimize the loss

For logistic regression, loss function is **convex**

- A convex function has just one minimum
- Gradient descent starting from any point is guaranteed to find the minimum
    - (Loss for neural networks is non-convex)

Gradients

The **gradient** of a function of many variables is a vector pointing in the direction of the greatest increase in a function.

Gradient Descent: Find the gradient of the loss function at the current point and move in the **opposite** direction.

How much do we move in that direction?

- The value of the gradient (slope in our example) $\frac{d}{dw}L(f(x;w),y)$
  - weighted by a learning rate η

- Higher learning rate means move w faster

$$w^{t+1} = w^t - \eta \frac{d}{dw}L(f(x;w),y)$$

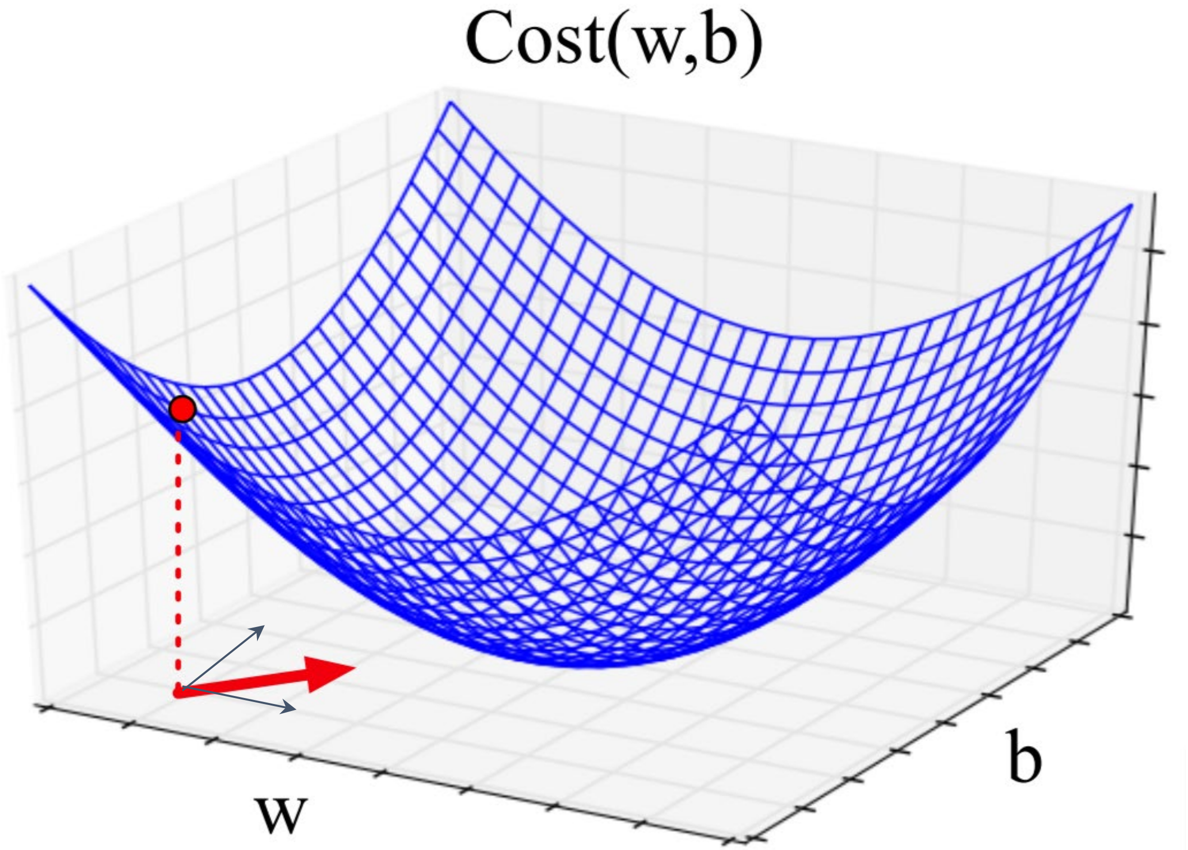
Now let's consider N dimensions

We want to know where in the N-dimensional space (of the N parameters that make up θ) we should move.

The gradient is just such a vector; it expresses the directional components of the sharpest slope along each of the N dimensions.

Imagine 2 dimensions, w and b

Visualizing the gradient vector at the red point
It has two dimensions shown in the x-y plane



Real gradients

Are much longer; lots and lots of weights

For each dimension w_i the gradient component i tells us the slope with respect to that variable.

- “How much would a small change in w_i influence the total loss function L?”
- We express the slope as a partial derivative ∂

The gradient is then defined as a vector of these partials.

The gradient

We'll represent $\hat{y}$ as $f(x; \theta)$ to make the dependence on θ more obvious:

$$\nabla_\theta L(f(x;\theta),y)) \;=\; \begin{bmatrix} \frac{\partial}{\partial w_1} L(f(x;\theta),y) \\ \frac{\partial}{\partial w_2} L(f(x;\theta),y) \\ \vdots \\ \frac{\partial}{\partial w_n} L(f(x;\theta),y) \end{bmatrix}$$

The final equation for updating θ based on the gradient is thus:

$$\theta_{t+1} \;=\; \theta_t - \eta \nabla L(f(x;\theta),y)$$

What are these partial derivatives for logistic regression?

The loss function

$$L_{\mathrm{CE}}(\hat{y},y) \;=\; -\left[y\log\sigma(w\cdot x+b)+(1-y)\log\left(1-\sigma(w\cdot x+b)\right)\right]$$

The elegant derivative of this function (see Section 5.10 for the derivation)

$$\begin{aligned}\frac{\partial L_{\mathrm{CE}}(\hat{y},y)}{\partial w_j} &= [\sigma(w\cdot x+b)-y]x_j \\ &= (\hat{y}-y)\mathbf{x}_j\end{aligned}$$

```
**function** STOCHASTIC GRADIENT DESCENT($L()$, $f()$, $x$, $y$) **returns** $\theta$
    # where: L is the loss function
    #        f is a function parameterized by $\theta$
    #        x is the set of training inputs $x^{(1)}, x^{(2)}, \ldots, x^{(m)}$
    #        y is the set of training outputs (labels) $y^{(1)}, y^{(2)}, \ldots, y^{(m)}$

$\theta \leftarrow 0$
**repeat** til done
   For each training tuple $(x^{(i)}, y^{(i)})$ (in random order)
      1. Optional (for reporting):                    # How are we doing on this tuple?
         Compute $\hat{y}^{(i)} = f(x^{(i)};\theta)$             # What is our estimated output $\hat{y}$?
         Compute the loss $L(\hat{y}^{(i)}, y^{(i)})$       # How far off is $\hat{y}^{(i)}$) from the true output $y^{(i)}$?
      2. $g \leftarrow \nabla_{\theta} L(f(x^{(i)};\theta), y^{(i)})$         # How should we move $\theta$ to maximize loss?
      3. $\theta \leftarrow \theta - \eta\, g$                        # Go the other way instead
return $\theta$
```

Hyperparameters

The learning rate η is a **hyperparameter**

- too high: the learner will take big steps and overshoot
- too low: the learner will take too long

Hyperparameters:

- Briefly, a special kind of parameter for an ML model
- Instead of being learned by algorithm from supervision (like regular parameters), they are chosen by algorithm designer.

Mini-batch training

Gradient descent computes the loss over the entire dataset. That can be slow.

More common to compute gradient over batches of training instances.

Batch training: entire dataset
Mini-batch training: m examples (512, or 1024)

Overfitting

A model that perfectly match the training data has a problem.

It will also **overfit** to the data, modeling noise

- A random word that perfectly predicts y (it happens to only occur in one class) will get a very high weight.
- Failing to generalize to a test set without this word.

A good model should be able to **generalize**

Regularization

A solution for overfitting

Add a **regularization** term R(θ) to the loss function (for now written as maximizing logprob rather than minimizing loss)

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{m} \log P(y^{(i)}|x^{(i)}) - \alpha R(\theta)$$

Idea: choose an R(θ) that penalizes large weights

- fitting the data well with lots of big weights not as good as fitting the data a little less well, with small weights

L2 regularization (ridge regression)

The sum of the squares of the weights

$$R(\theta) \;=\; ||\theta||_2^2 = \sum_{j=1}^{n} \theta_j^2$$

L2 regularized objective function:

$$\hat{\theta} \;=\; \underset{\theta}{\operatorname{argmax}} \left[\sum_{i=1}^{m} \log P(y^{(i)}|x^{(i)}) \right] - \alpha \sum_{j=1}^{n} \theta_j^2$$

Multinomial Logistic Regression

Often we need more than 2 classes

- Positive/negative/neutral
- Classify emergency SMSs into different actionable classes

If >2 classes we use **multinomial logistic regression**
= Softmax regression
= Multinomial logit
= (defunct names : Maximum entropy modeling or MaxEnt)

So "logistic regression" will just mean binary (2 output classes)

Multinomial Logistic Regression

The probability of everything must still sum to 1
P(positive|doc) + P(negative|doc) + P(neutral|doc) = 1

Need a generalization of the sigmoid called the **softmax**

- Takes a vector $z = [z_1, z_2, ..., z_k]$ of k arbitrary values
- Outputs a probability distribution
- each value in the range $[0,1]$
- all the values summing to 1

We'll discuss it more when we talk about neural networks

One-hot representation

Gold labels – one-hot representations

[0, 0, ..., 1, 0, 0]

Predicted values – vector of class probabilities

[0.1, 0.05, ..., .08, 0, 0.07]

Sigmoid → softmax

$$\frac{1}{1+e^{-z}} \longrightarrow \frac{e^{z_j}}{\sum_{c=1}^{k} e^{z_c}}$$

The **softmax** function

- Turns a vector $z = [z_1, z_2, \ldots, z_k]$ of k arbitrary values (logits) into probabilities

$$\text{softmax}(z_i) = \frac{\exp(z_i)}{\sum_{j=1}^{k} \exp(z_j)} \quad 1 \leq i \leq k$$

- The denomina $\sum_{i=1}^{k} e^{z_i}$ is used to normalize all the values into probabilities

$$\text{softmax}(z) = \left[\frac{\exp(z_1)}{\sum_{i=1}^{k} \exp(z_i)}, \frac{\exp(z_2)}{\sum_{i=1}^{k} \exp(z_i)}, \ldots, \frac{\exp(z_k)}{\sum_{i=1}^{k} \exp(z_i)} \right]$$

softmax: a generalization of sigmoid

- For a vector z of dimensionality k, the softmax is:

$$\text{softmax}(z) = \left[\frac{\exp(z_1)}{\sum_{i=1}^{k}\exp(z_i)}, \frac{\exp(z_2)}{\sum_{i=1}^{k}\exp(z_i)}, \ldots, \frac{\exp(z_k)}{\sum_{i=1}^{k}\exp(z_i)}\right]$$

$$\text{softmax}(z_i) = \frac{\exp(z_i)}{\sum_{j=1}^{k}\exp(z_j)} \quad 1 \leq i \leq k$$

Example:

$$z = [0.6, 1.1, -1.5, 1.2, 3.2, -1.1]$$

$$\text{softmax}(z) = [0.055, 0.090, 0.006, 0.099, 0.74, 0.010]$$

Components of a probabilistic machine learning classifier

Given m input/output pairs $(x^{(i)}, y^{(i)})$:

1. A **feature representation** for the input. For each input observation $x^{(i)}$, a vector of features $[x_1, x_2, \ldots, x_n]$. Feature j for input $x^{(i)}$ is x_j, more completely $x_1^{(i)}$, or sometimes $f_j(x)$.

2. A **classification function** that computes $\hat{y}$ the estimated class, via $p(y|x)$, like the **sigmoid** or **softmax** functions

3. An **objective function** for learning, like **cross-entropy loss**

4. An algorithm for **optimizing** the objective function: **stochastic gradient descent**

Next class:

- Neural Network Basics