

# Language Modeling III: Transformers

CSE 5525: Foundations of Speech and Language Processing  
<https://shocheen.github.io/cse-5525-fall-2025/>



**THE OHIO STATE UNIVERSITY**

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Sachin Kumar (kumar.1145@osu.edu)

# Logistics

- Homework 2 is due date next week.
  - Any thoughts, questions, concerns?
- Final project: have you formed teams already?
  - A project proposal will be due first week of October.
  - Look at last year's course website for example projects, will also post other ideas on Canvas: <https://shocheen.github.io/cse-5525-fall-2025/>

# Recap

- Feedforward Neural Language Model
  - Need to make unreasonable assumptions and lose information from the long context
- Recurrent Neural Network (RNN)
  - Infinitely long context in theory --- hard to train (exploding/vanishing gradients), difficult to parallelize, and could be infeasible (memorize a variable length sequence in a fixed length vector).
  - Encoder-decoder architecture
- RNN + Attention
  - Solves the last issue, still hard to train efficiently (on GPUs).
- Attention is all you need [will continue today]
  - Transformer Architecture

# Transformers

- Replace the linear part of RNNs with **self-attention**
- Introduce **residual connections + layernorm** to improve gradient flow (avoid gradient vanishing issues)
- Introduce **positional embeddings** to encode sequential order

# Outline



Self-Attention



Transformer Encoder







Transformer Decoder



Language Modeling With Transformers

# Outline

-  Self-Attention
-  Transformer Encoder
-  Transformer Decoder
-  Language Modeling With Transformers

# Defining Self-Attention

- **Terminology:**

- **Query**: to match others
- **Key**: to be matched
- **Value**: information to be extracted

- **Definition:** Given a set of vector **values**, and a vector **query**, *attention* is a technique to compute a weighted sum of the **value**, dependent on the **keys**.

$q$ : query (to match others)

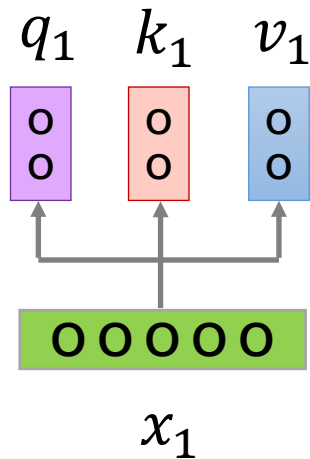
$$q_t = W^q x_t$$

$k$ : key (to be matched)

$$k_t = W^k x_t$$

$v$ : value (information to be extracted)

$$v_t = W^v x_t$$



The



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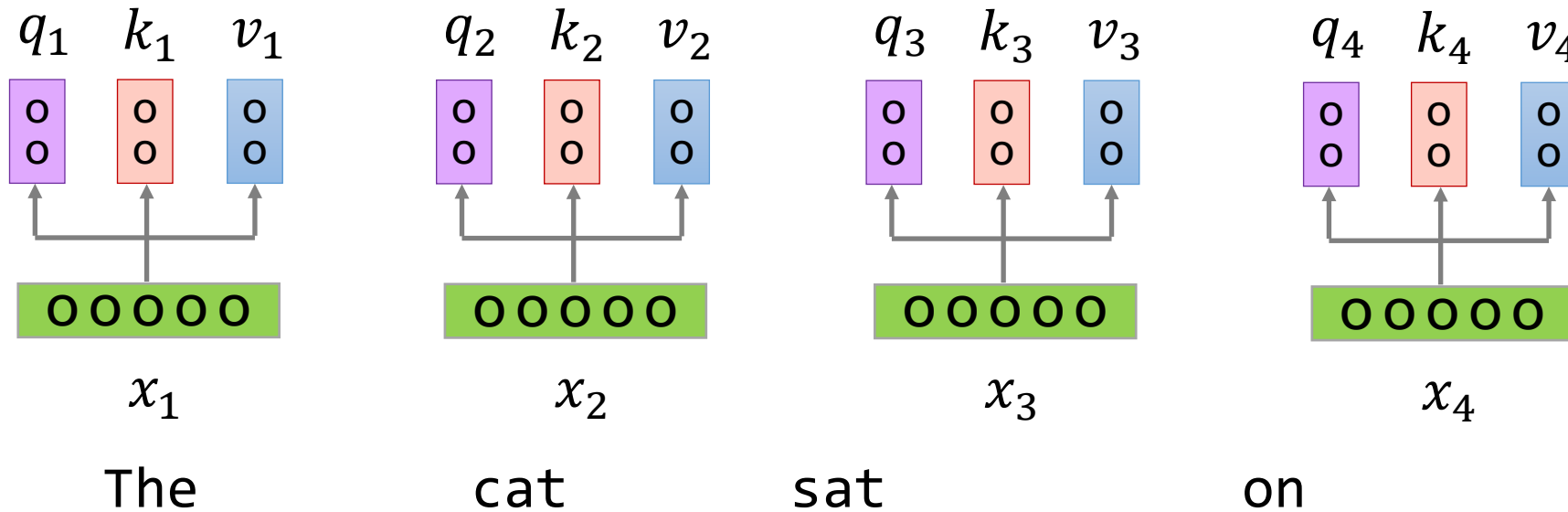
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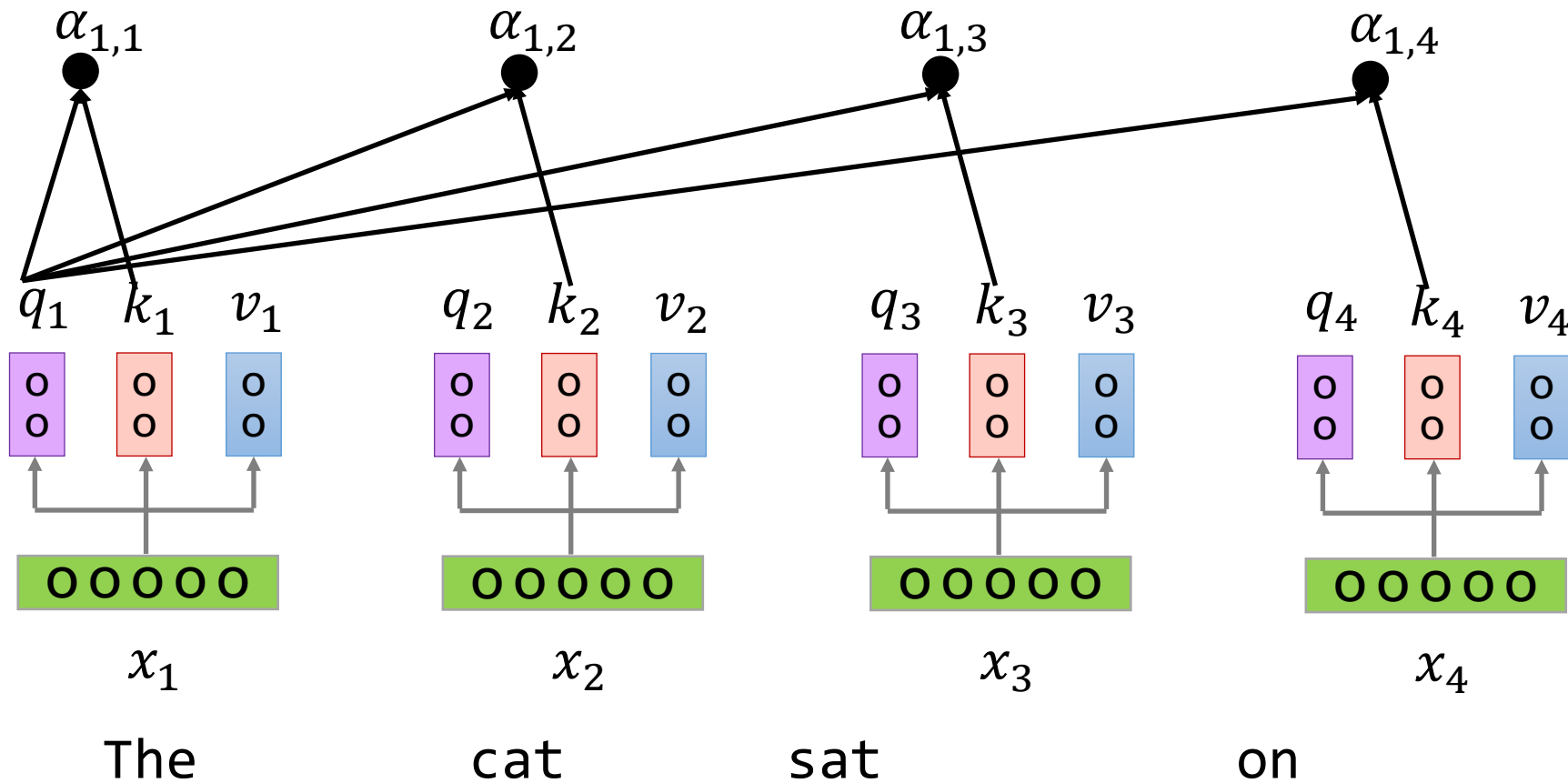
$$\alpha_{1,t} = \underbrace{q^1 \cdot k^t / \alpha}_{\text{Scaled dot product}}$$

$q$ : query (to match others)

$k$ : key (to be matched)

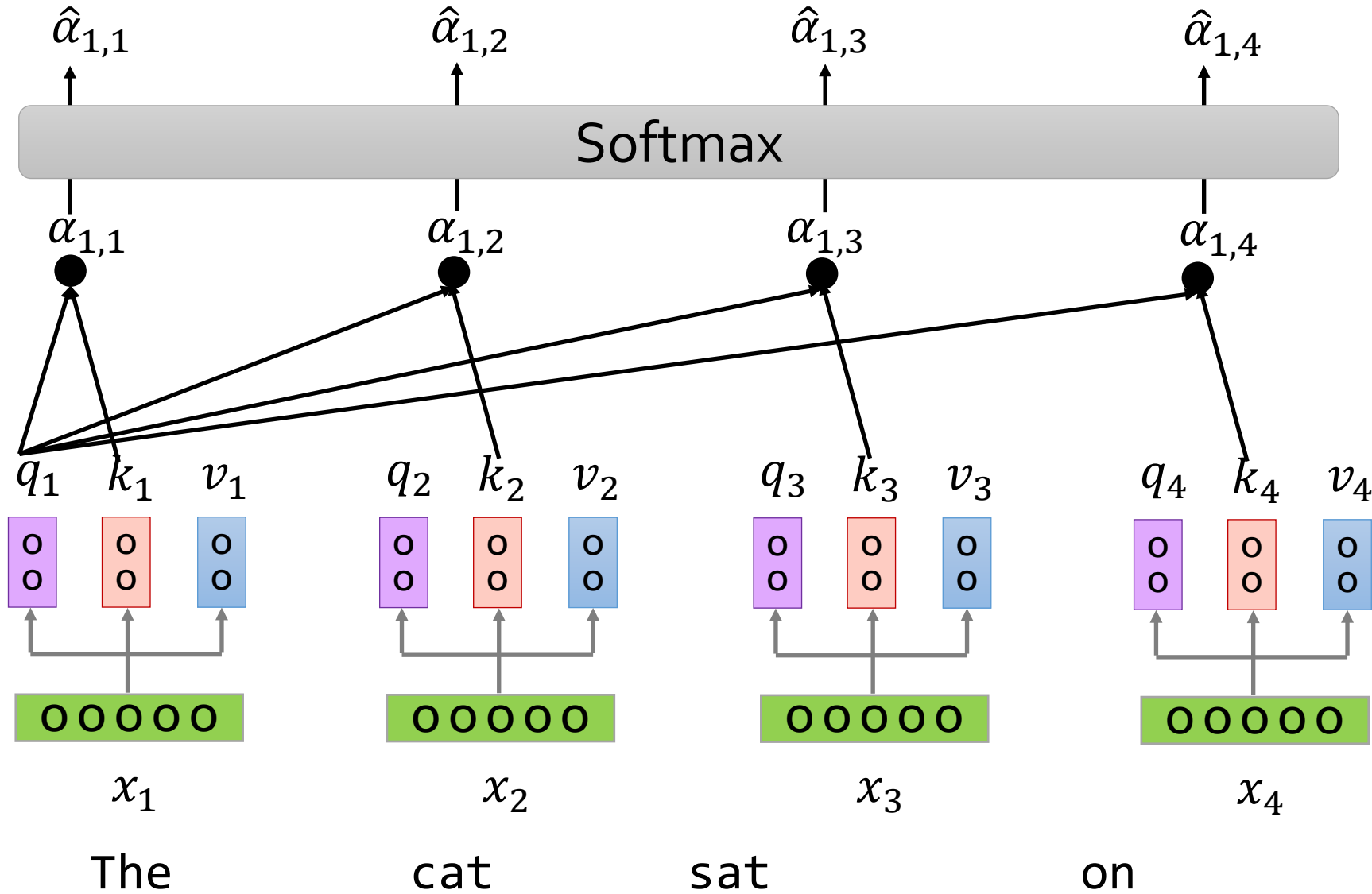
$v$ : value (information to be extracted)

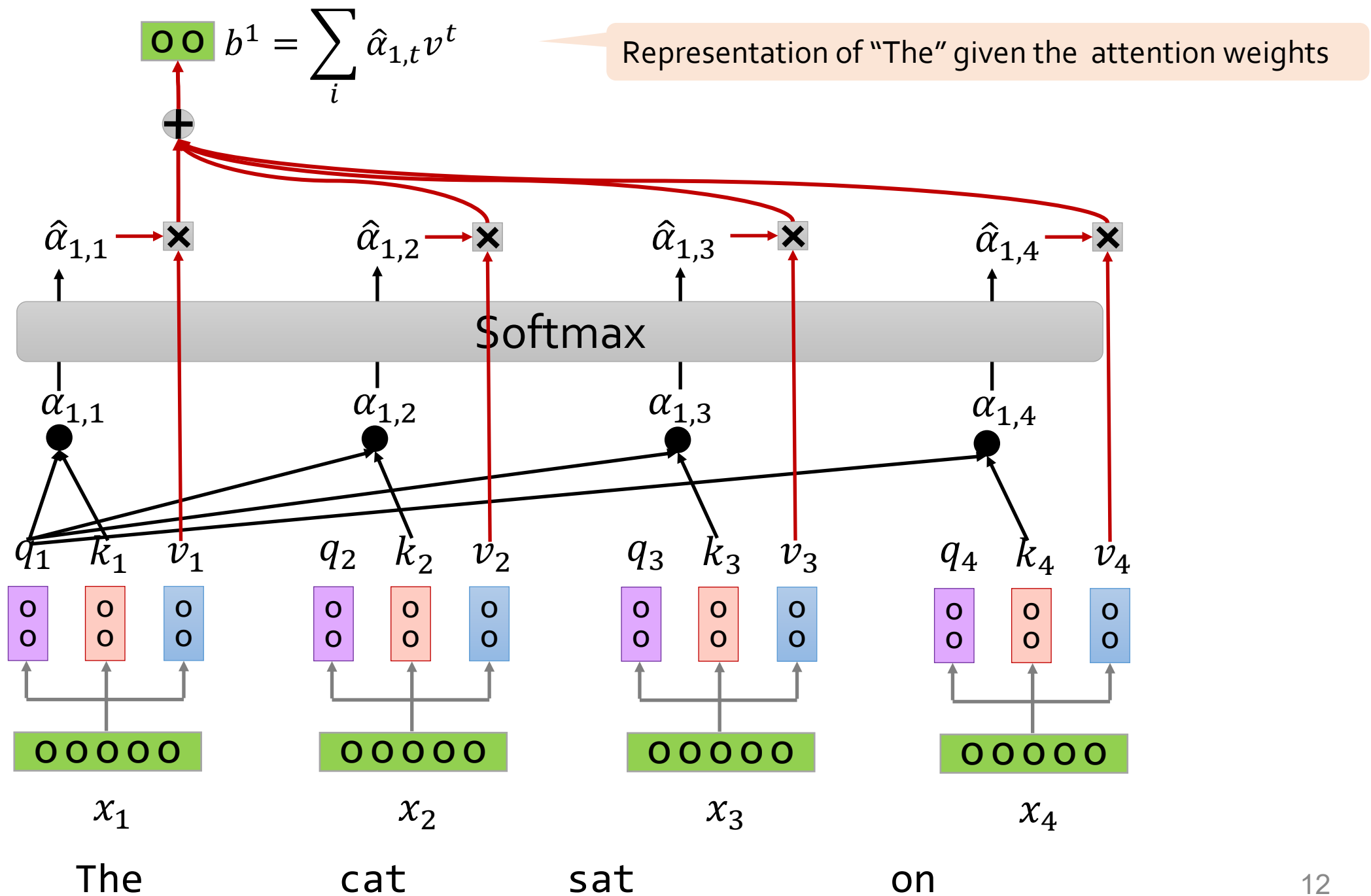
How much should "The" attend to other positions?

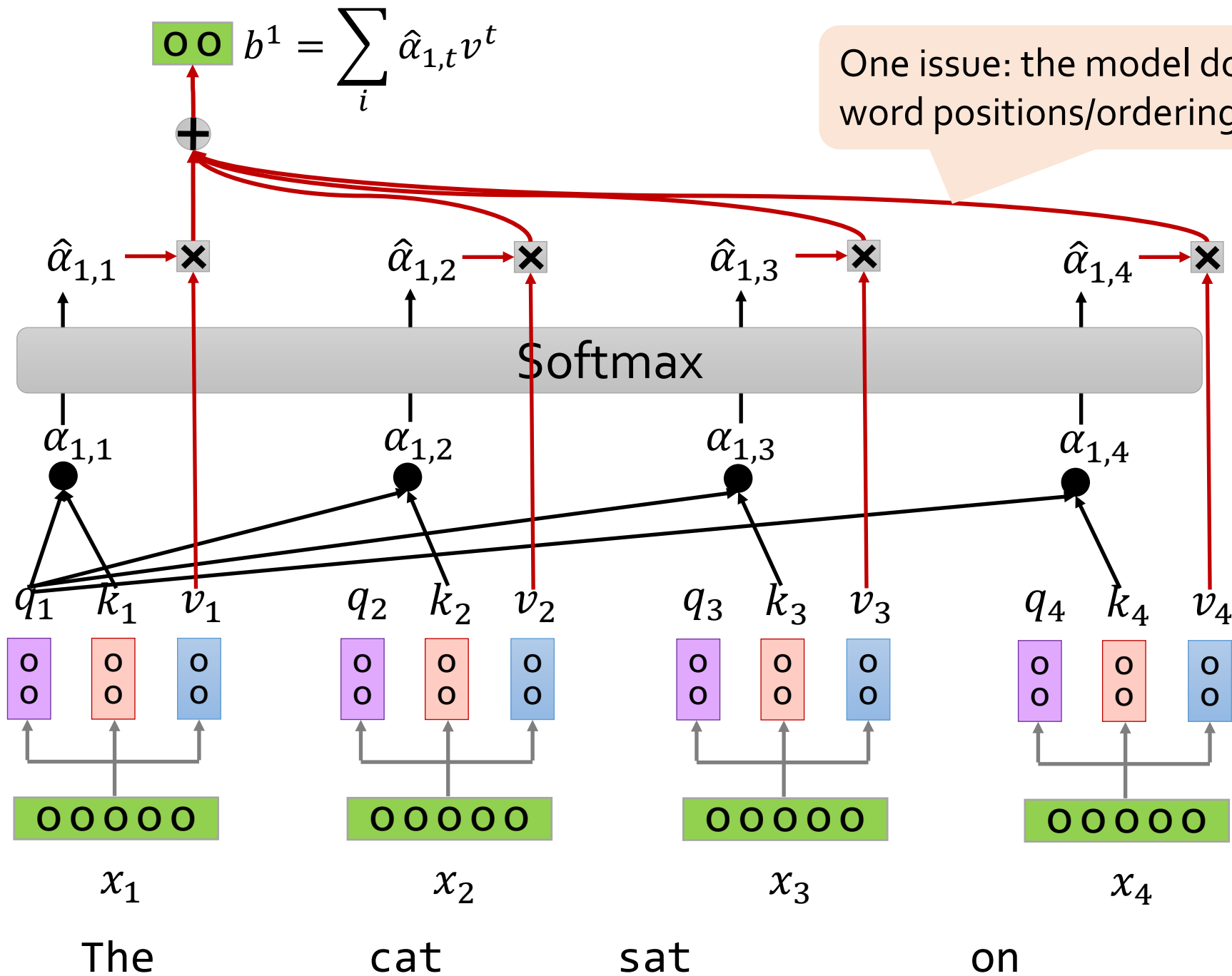


$$\sigma(z)_t = \frac{\exp(z_t)}{\sum_j \exp(z_j)}$$

How much should "The" attend to other positions?







# Self-Attention

**Step 1:** Our Self-Attention Head I has just 3 weight matrices  $W_q$ ,  $W_k$ ,  $W_v$  in total. These same 3 weight matrices are multiplied by each  $x_i$  to create all vectors:

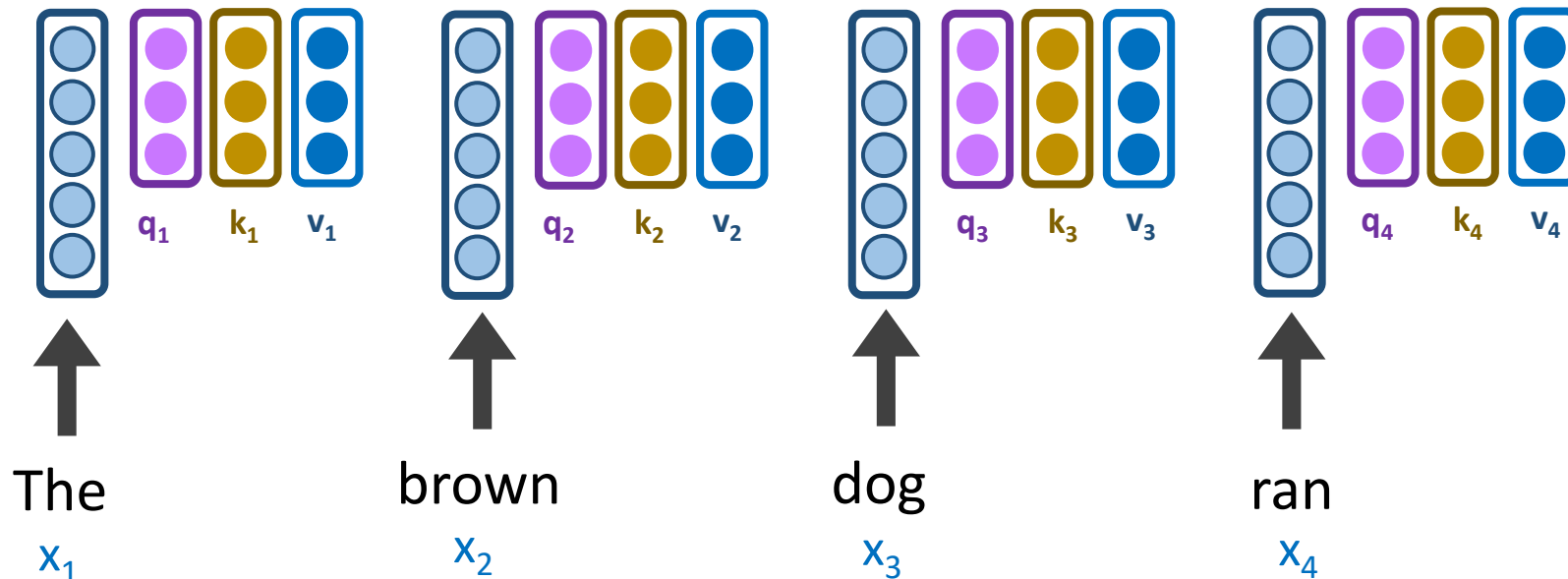
$$q_i = w_q x_i$$

$$k_i = w_k x_i$$

$$v_i = w_v x_i$$

Under the hood, each  $x_i$  has 3 small, associated vectors. For example,  $x_1$  has:

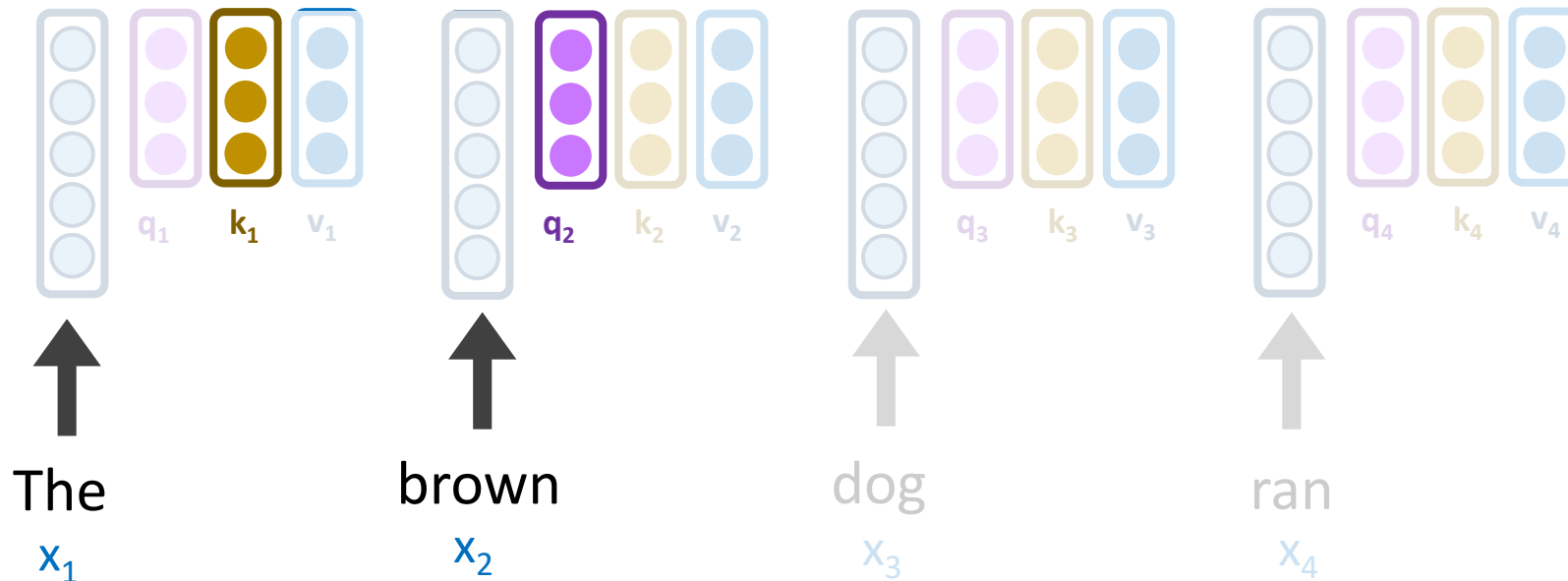
- Query  $q_1$
- Key  $k_1$
- Value  $v_1$



# Self-Attention

**Step 2:** For word  $x_2$ , let's calculate the scores  $s_1, s_2, s_3, s_4$ , which represent how much attention to pay to each respective "word"  $v_i$

$$s_1 = q_2 \cdot k_1 = 92$$

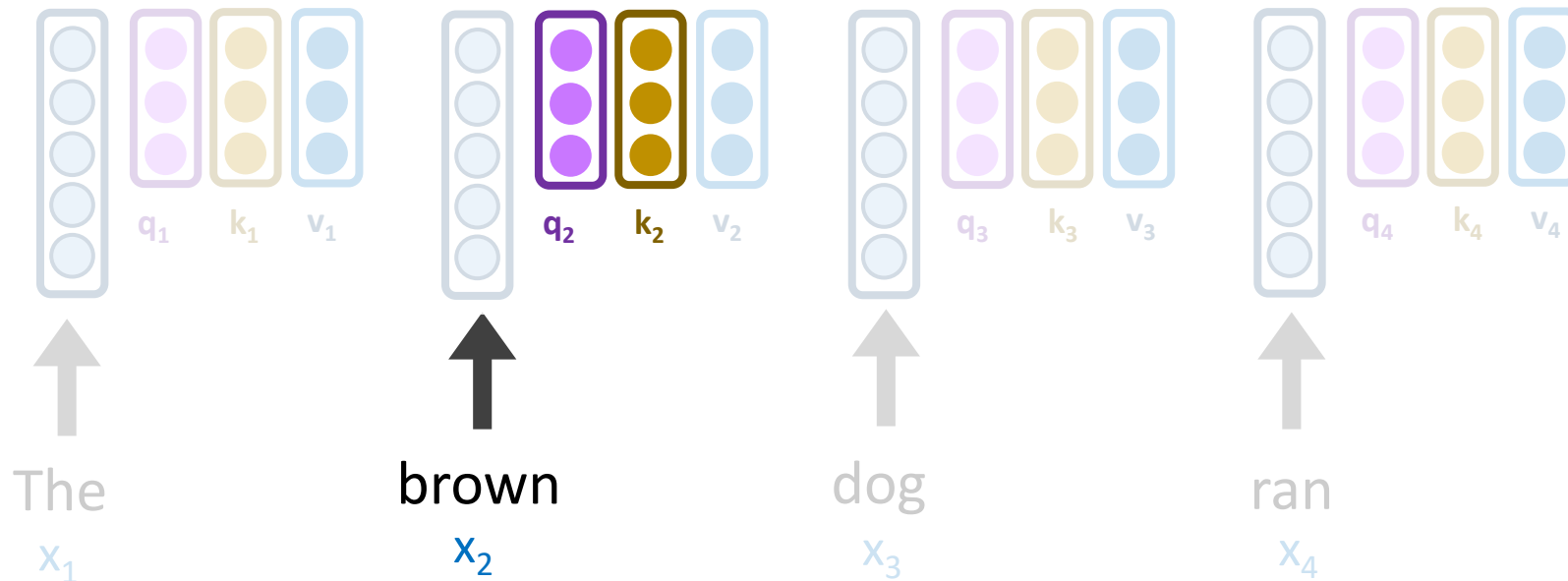


# Self-Attention

**Step 2:** For word  $x_2$ , let's calculate the scores  $s_1, s_2, s_3, s_4$ , which represent how much attention to pay to each respective "word"  $v_i$

$$s_2 = q_2 \cdot k_2 = 124$$

$$s_1 = q_2 \cdot k_1 = 92$$





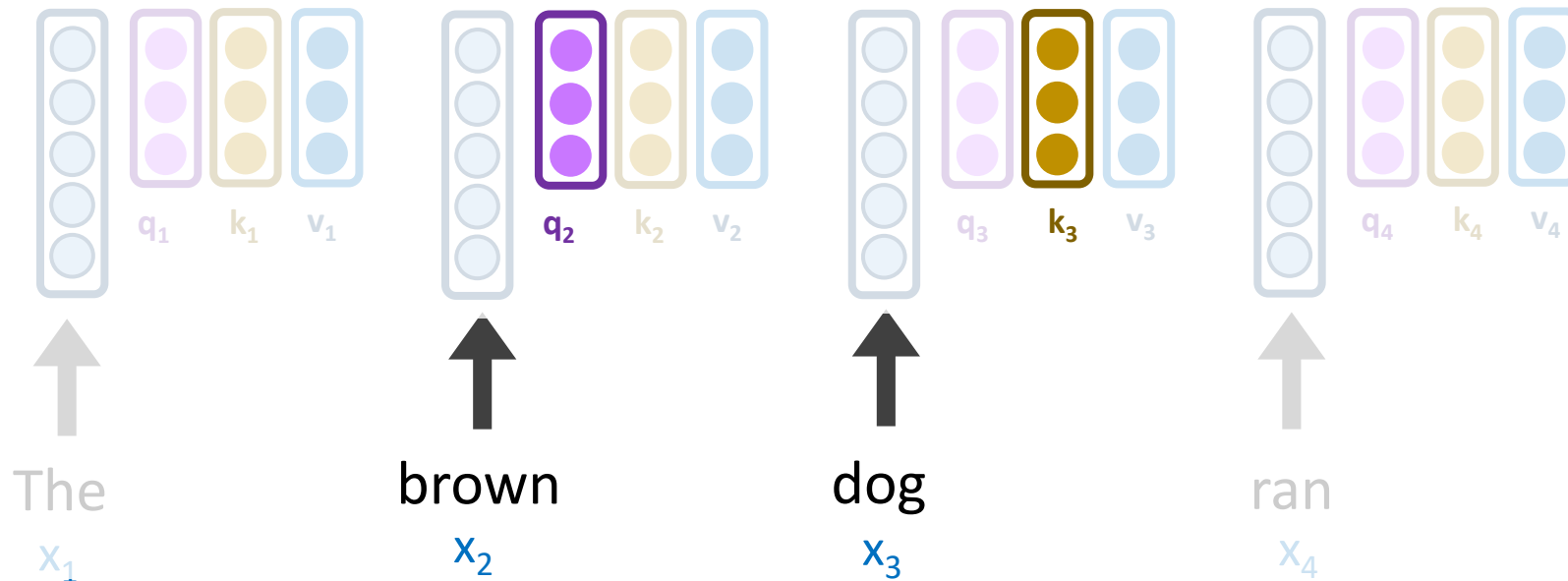
# Self-Attention

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$$s_3 = q_2 \cdot k_3 = 22$$

$$s_2 = q_2 \cdot k_2 = 124$$

$$s_1 = q_2 \cdot k_1 = 92$$



# Self-Attention

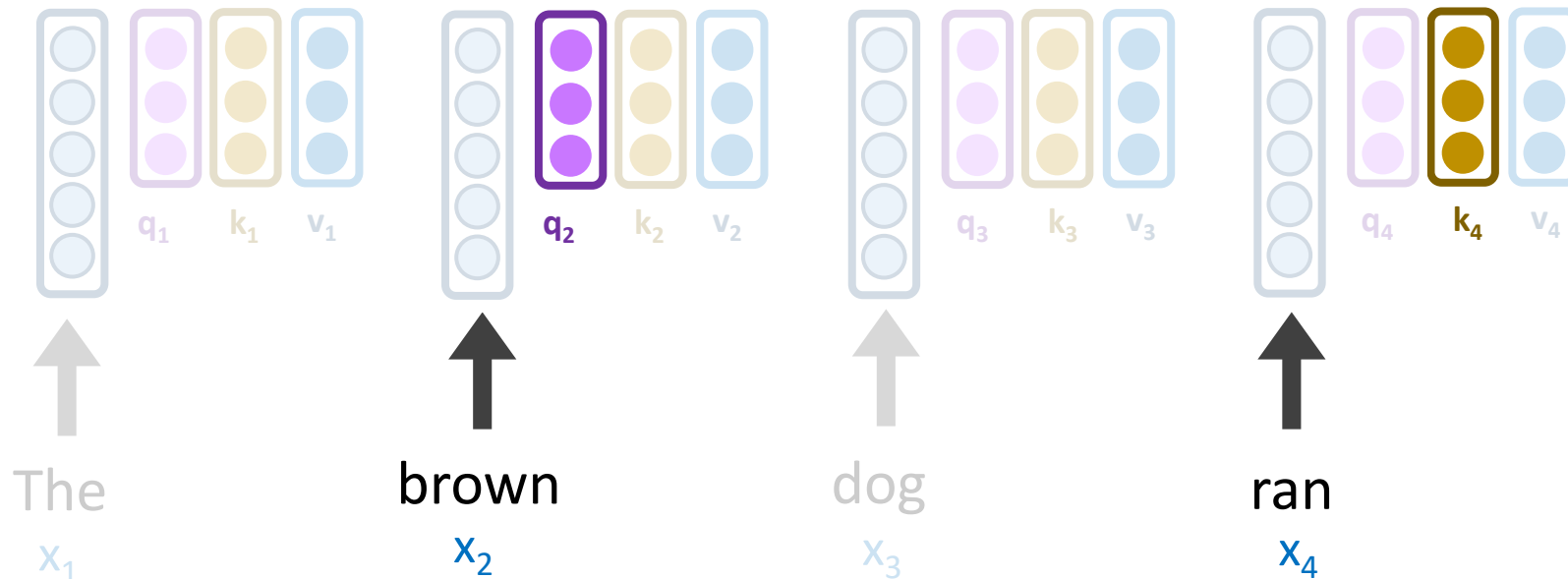
**Step 2:** For word  $x_2$ , let's calculate the scores  $s_1, s_2, s_3, s_4$ , which represent how much attention to pay to each respective "word"  $v_i$

$$s_4 = q_2 \cdot k_4 = 8$$

$$s_3 = q_2 \cdot k_3 = 22$$

$$s_2 = q_2 \cdot k_2 = 124$$

$$s_1 = q_2 \cdot k_1 = 92$$



# Self-Attention

**Step 3:** Our scores  $s_1, s_2, s_3, s_4$  don't sum to 1. Let's divide by  $\sqrt{\text{len}(k_i)}$  and **softmax** it

$$s_4 = q_2 \cdot k_4 = 8$$

$$a_4 = \sigma(s_4/8) = 0$$

$$s_3 = q_2 \cdot k_3 = 22$$

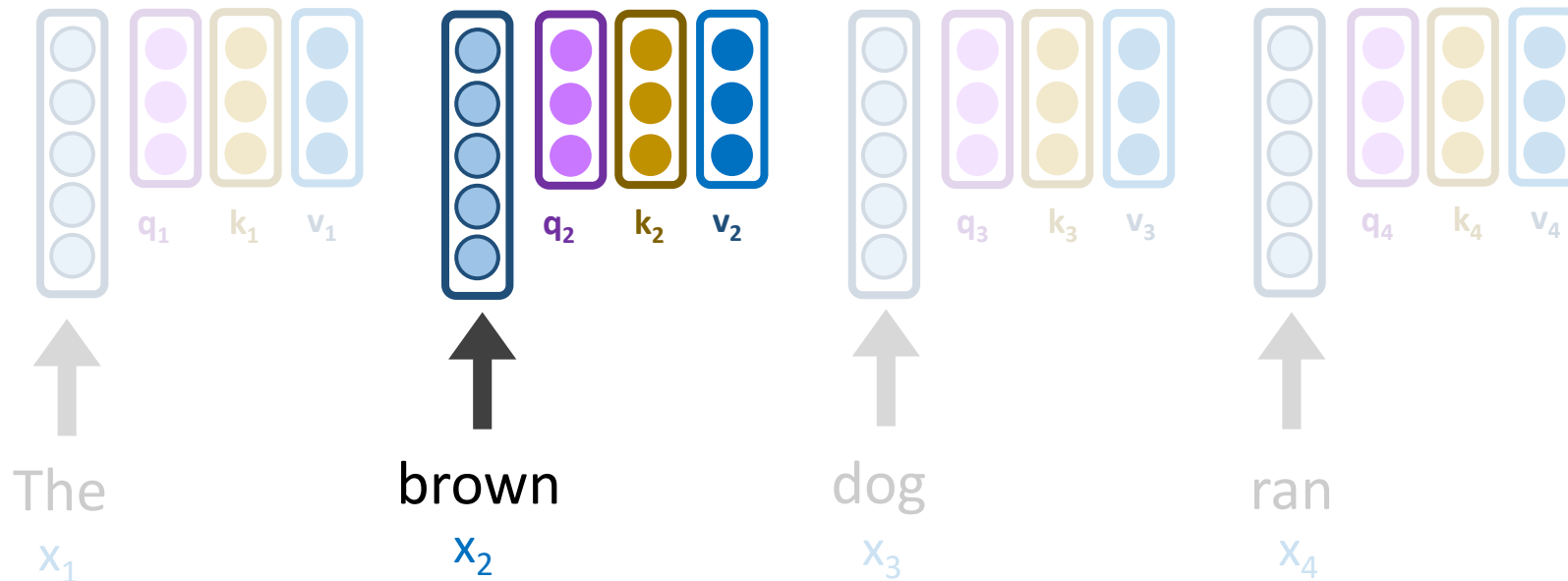
$$a_3 = \sigma(s_3/8) = .01$$

$$s_2 = q_2 \cdot k_2 = 124$$

$$a_2 = \sigma(s_2/8) = .91$$

$$s_1 = q_2 \cdot k_1 = 92$$

$$a_1 = \sigma(s_1/8) = .08$$



# Self-Attention

**Step 3:** Our scores  $s_1, s_2, s_3, s_4$  don't sum to 1. Let's divide by  $\sqrt{\text{len}(k_i)}$  and **softmax** it

$$s_4 = \mathbf{q}_2 \cdot \mathbf{k}_4 = 8$$

$$s_3 = \mathbf{q}_2 \cdot \mathbf{k}_3 = 22$$

$$s_2 = \mathbf{q}_2 \cdot \mathbf{k}_2 = 124$$

$$s_1 = \mathbf{q}_2 \cdot \mathbf{k}_1 = 92$$

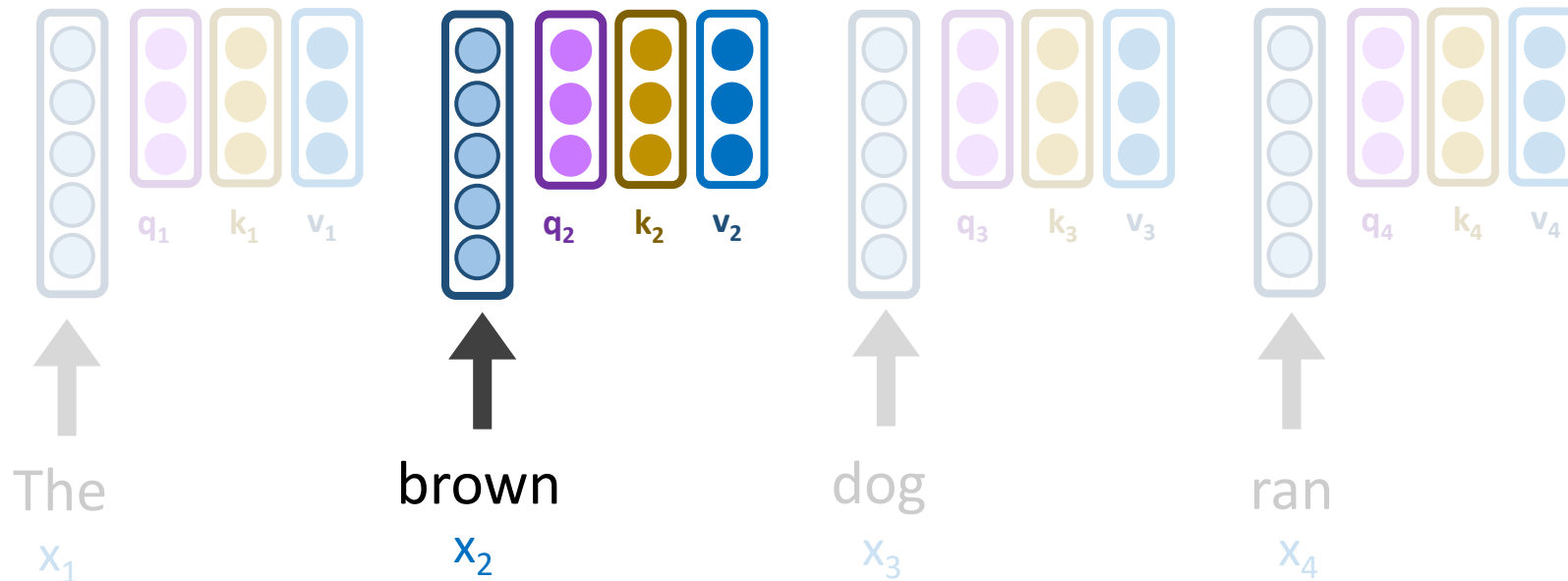
$$\mathbf{a}_4 = \sigma(s_4/8) = 0$$

$$\mathbf{a}_3 = \sigma(s_3/8) = .01$$

$$\mathbf{a}_2 = \sigma(s_2/8) = .91$$

$$\mathbf{a}_1 = \sigma(s_1/8) = .08$$

Instead of these  $\mathbf{a}_i$  values directly weighting our original  $\mathbf{x}_i$  word vectors, they directly weight our  $\mathbf{v}_i$  vectors.

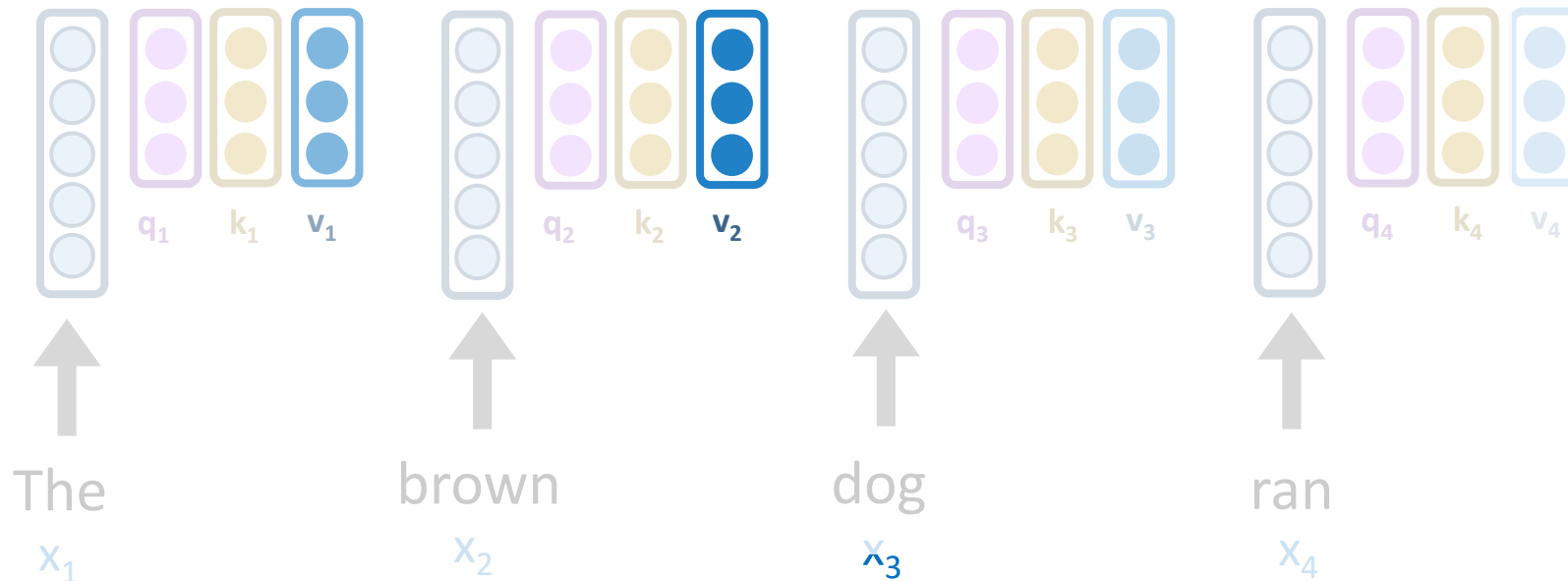


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**Step 4:** Let's weight our  $\mathbf{v}_i$  vectors and simply sum them up!

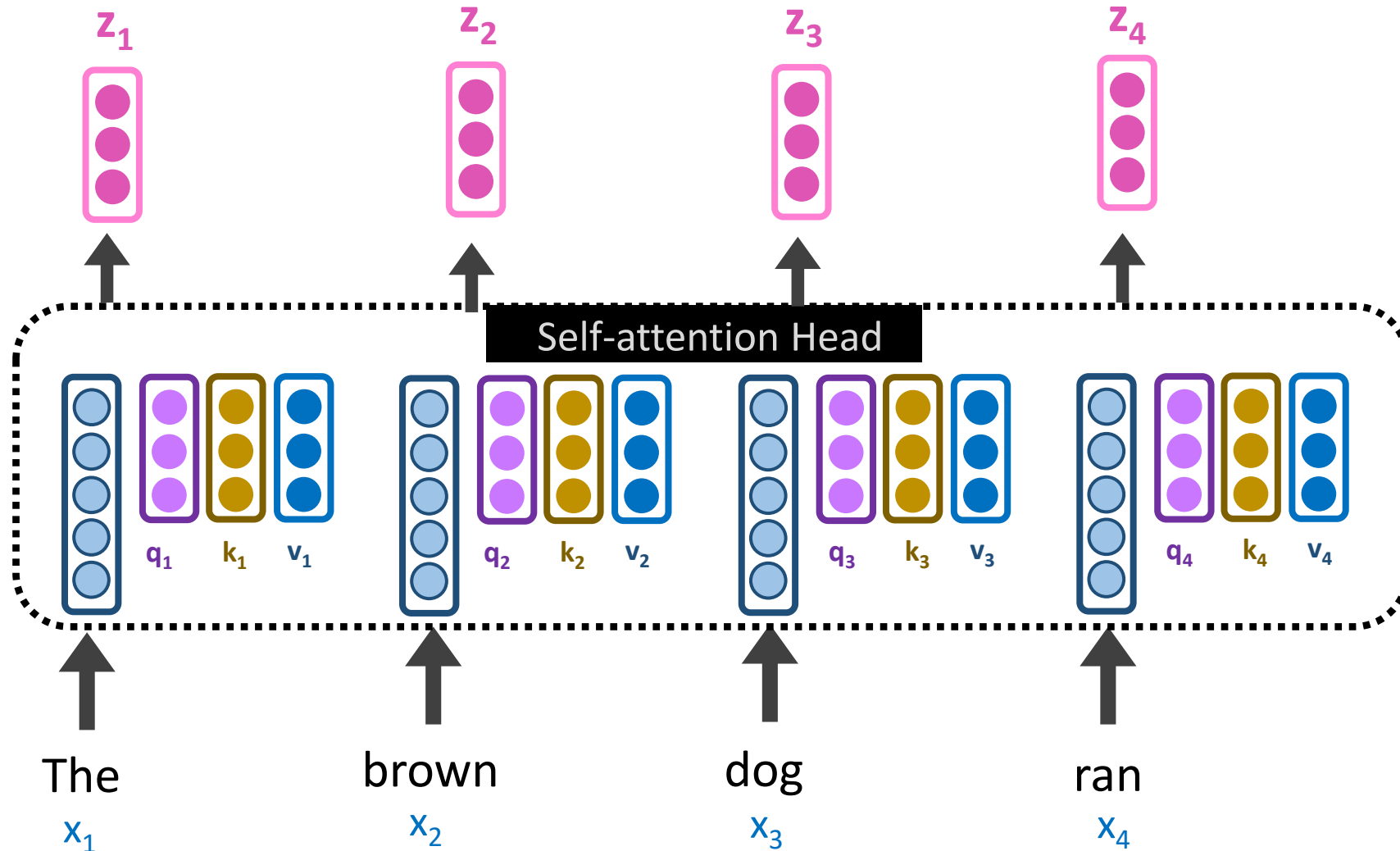


$$\begin{aligned} \mathbf{z}_2 &= \mathbf{a}_1 \cdot \mathbf{v}_1 + \mathbf{a}_2 \cdot \mathbf{v}_2 + \mathbf{a}_3 \cdot \mathbf{v}_3 + \mathbf{a}_4 \cdot \mathbf{v}_4 \\ &= 0.08 \cdot \mathbf{v}_1 + 0.91 \cdot \mathbf{v}_2 + 0.01 \cdot \mathbf{v}_3 + 0 \cdot \mathbf{v}_4 \end{aligned}$$



# Self-Attention

Tada! Now we have great, new representations  $z_i$  via a self-attention head

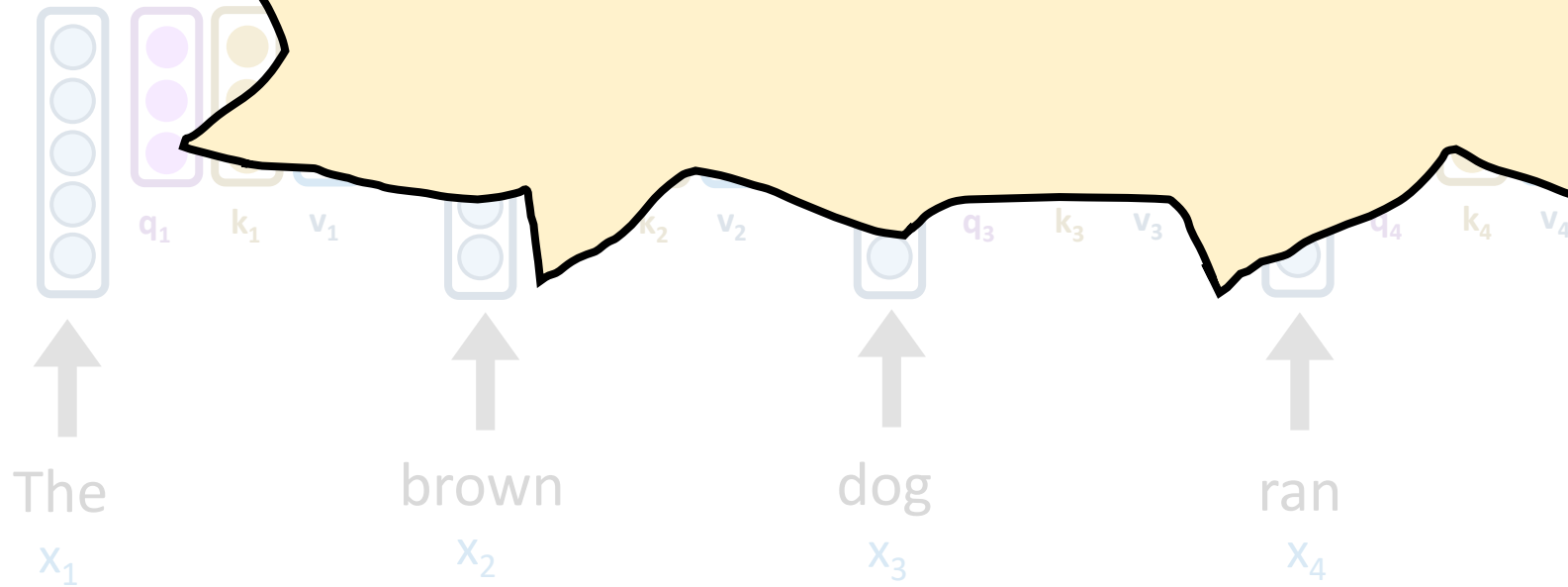


# Self-Attention

Tada! Now

**Takeaway:**

**Self-Attention** is powerful; allows us to create great, context-aware representations



# Self-Attention

$$\mathbf{b} = \text{softmax}\left(\frac{\mathbf{Q}\mathbf{K}^T}{\alpha}\right)\mathbf{V}$$



hardmaru  
@hardmaru



The most important formula in deep learning after 2018

## Self-Attention

**What is self-attention?** Self-attention calculates a weighted average of feature representations with the weight proportional to a similarity score between pairs of representations. Formally, an input sequence of  $n$  tokens of dimensions  $d$ ,  $X \in \mathbf{R}^{n \times d}$ , is projected using three matrices  $W_Q \in \mathbf{R}^{d \times d_q}$ ,  $W_K \in \mathbf{R}^{d \times d_k}$ , and  $W_V \in \mathbf{R}^{d \times d_v}$  to extract feature representations  $Q$ ,  $K$ , and  $V$ , referred to as query, key, and value respectively with  $d_k = d_q$ . The outputs  $Q$ ,  $K$ ,  $V$  are computed as

$$Q = XW_Q, \quad K = XW_K, \quad V = XW_V. \quad (1)$$

So, self-attention can be written as,

$$S = D(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_q}}\right)V, \quad (2)$$

where softmax denotes a *row-wise* softmax normalization function. Thus, each element in  $S$  depends on all other elements in the same row.

9:08 PM · Feb 9, 2021 · Twitter Web App

553 Retweets 42 Quote Tweets 3,338 Likes



# Outline



Self-Attention



Transformer Encoder



Transformer Decoder



Language Modeling With Transformers

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Self-Attention



Transformer Encoder



Transformer Decoder

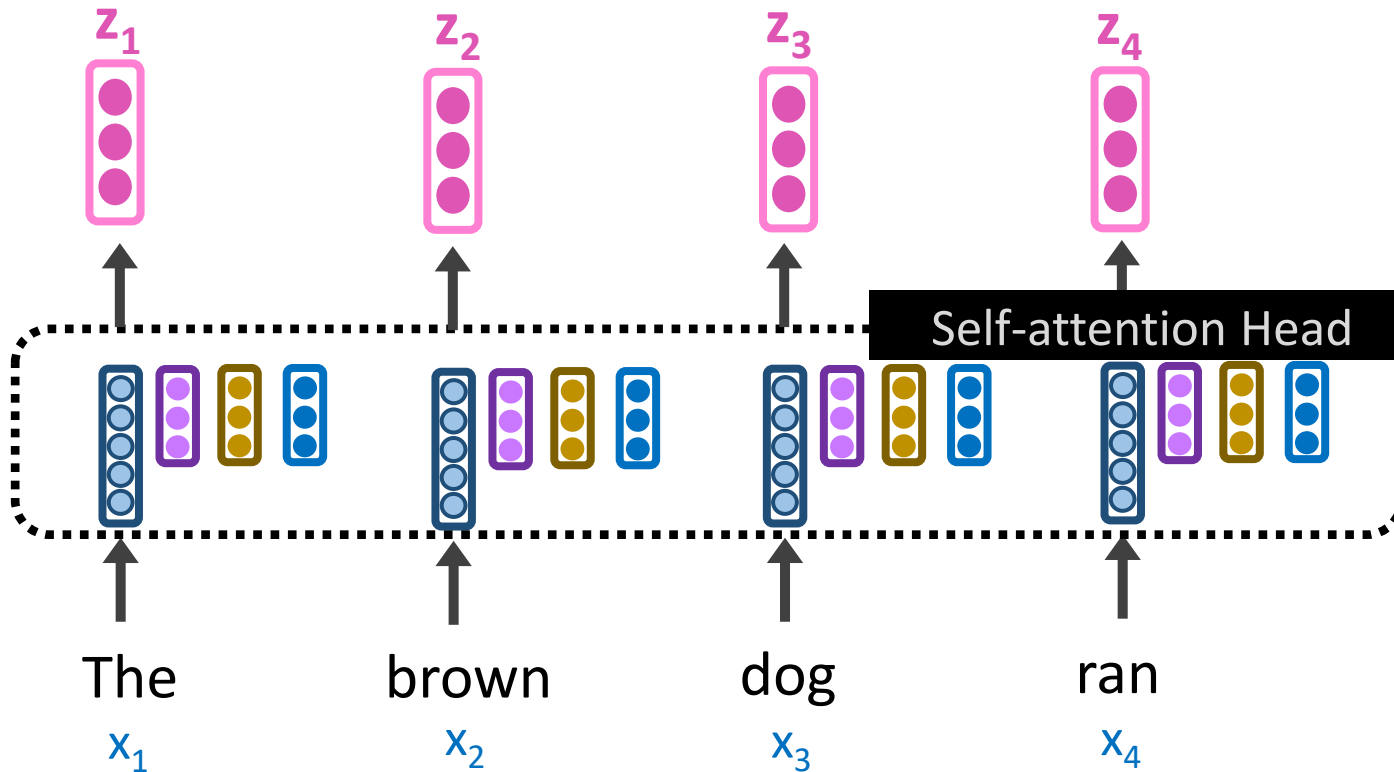


Language Modeling With Transformers

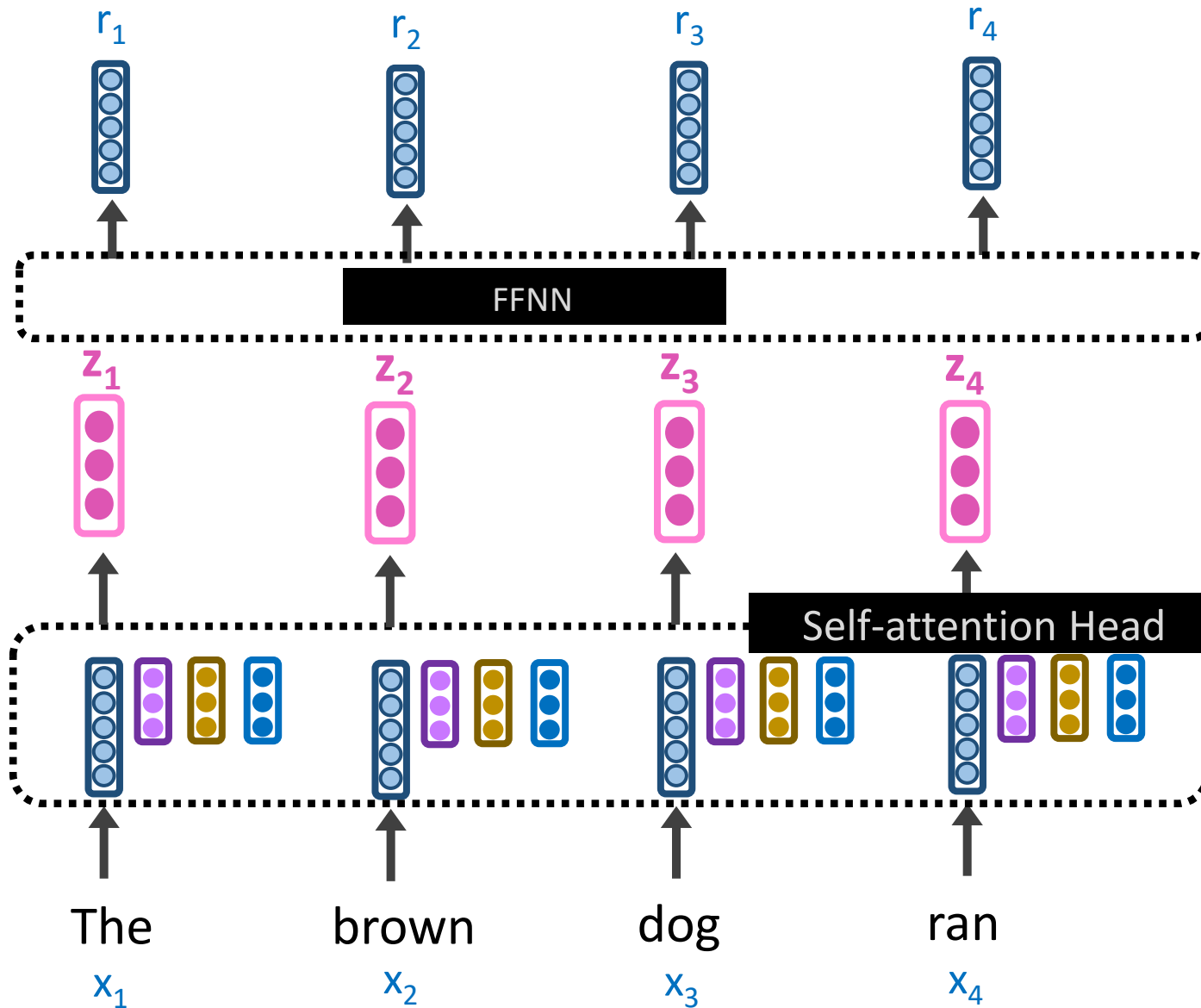
# Self-Attention

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Let's further pass each  $z_i$  through a FeedForward NN

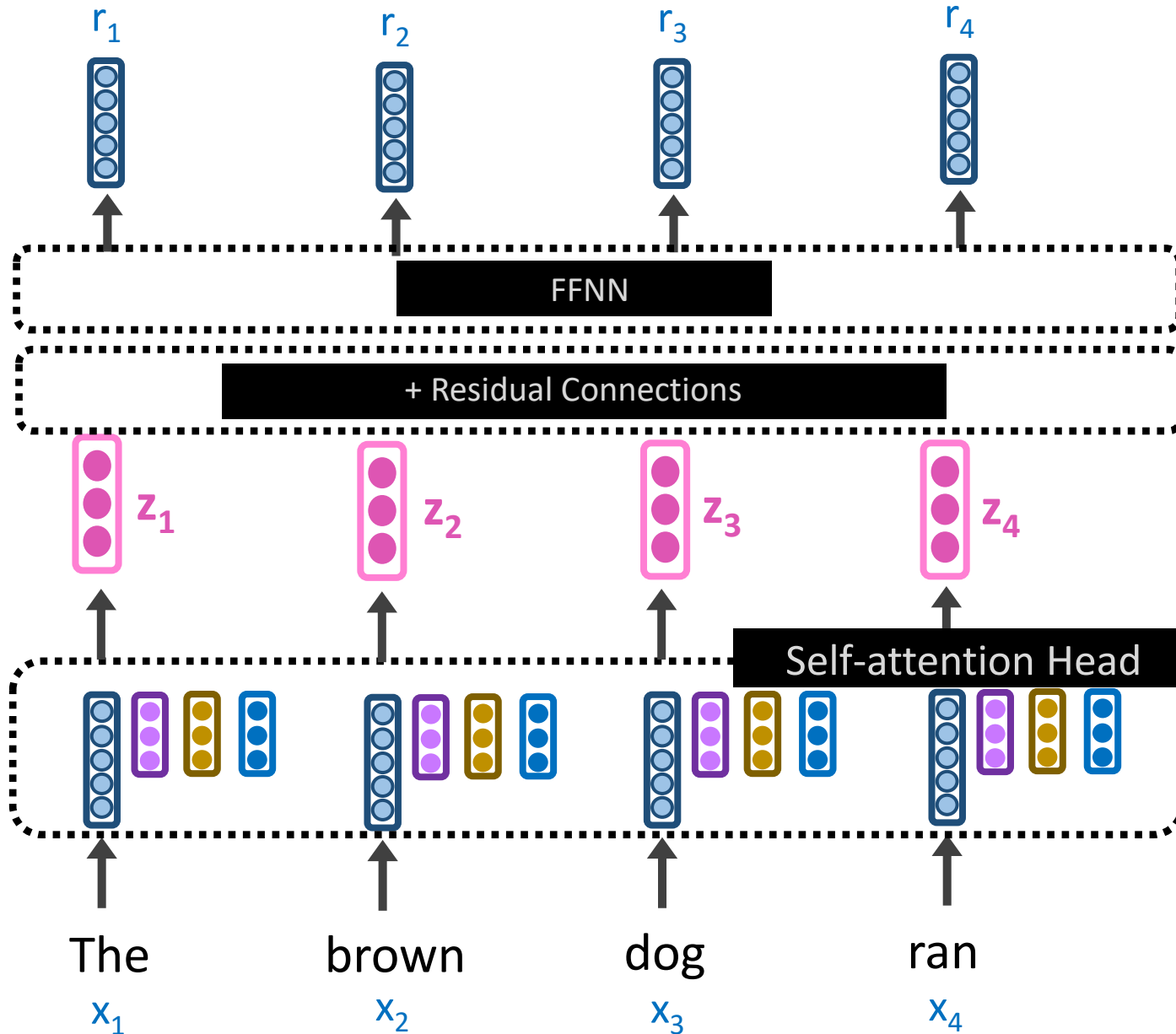


# Self-Attention + FFNN



Let's further pass each  $z_i$  through a Feed Forward NN

# Self-Attention + FFNN + Residual Connections

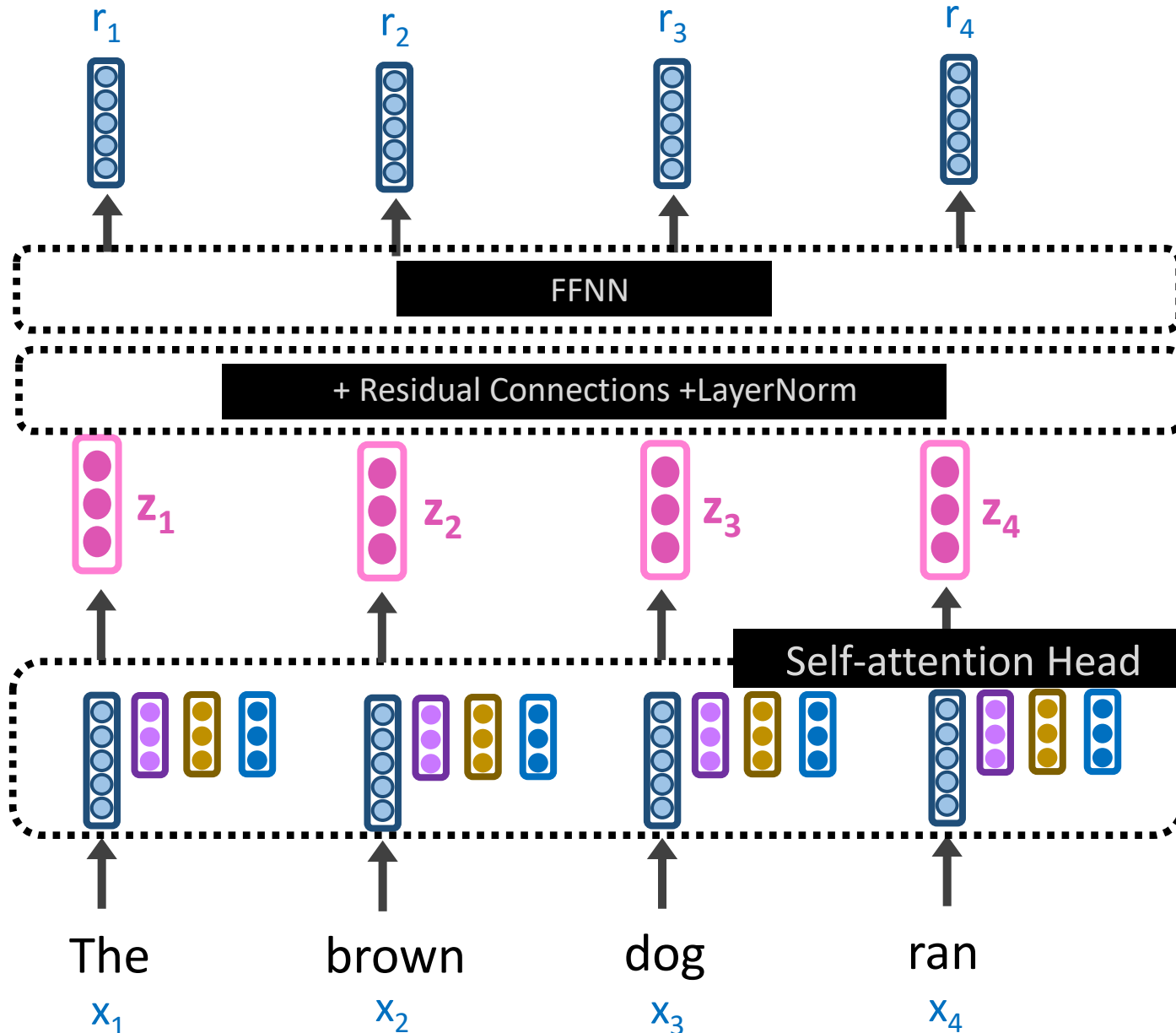


Let's further pass each  $z_i$  through a FFNN

We add a **residual connection** to help ensure relevant info is getting forward passed.

$$v = z + x$$

# Self-Attention + FFNN + Residual Connections



Let's further pass each  $z_i$  through a FFNN

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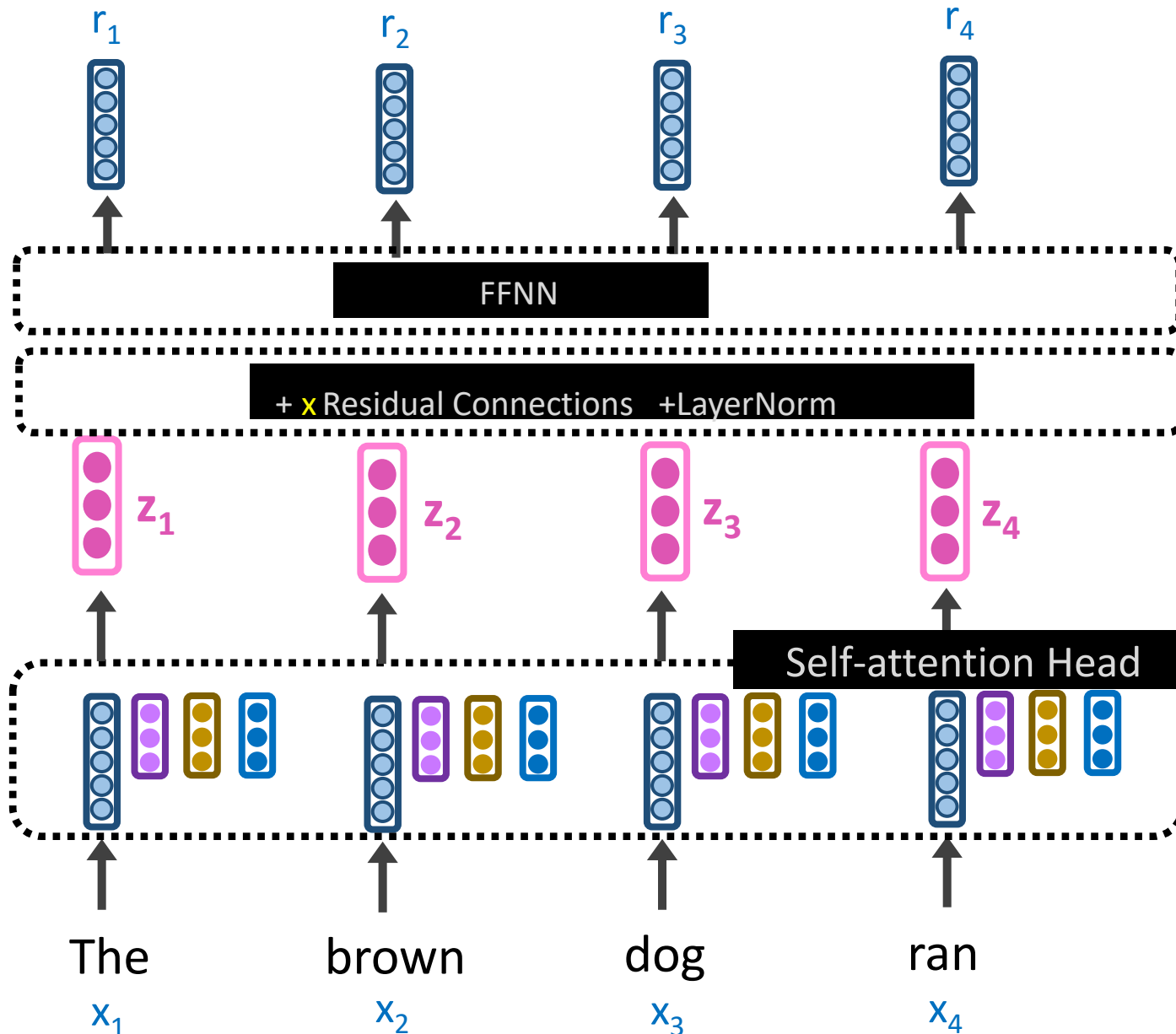
We perform **LayerNorm** to stabilize the network and allow for proper gradient flow. You should do this after the FFNN, too.

# Stabilizing Gradient Flow: Residual Connection and Layernorm

- Residual connection:  $y = f(x) + x$ 
  - $f$  might be a complex function and gives small gradients wrt  $x$ , adding  $x$  back to  $f(x)$  gives higher values of the gradient
- Layer Normalization (LayerNorm):
  - Another way to prevent vanishing gradients

$$y = \frac{x - \mathbb{E}[x]}{\sqrt{\text{Var}[x] + \epsilon}} * \gamma + \beta$$

# Self-Attention + FFNN



Let's further pass each  $z_i$  through a FFNN

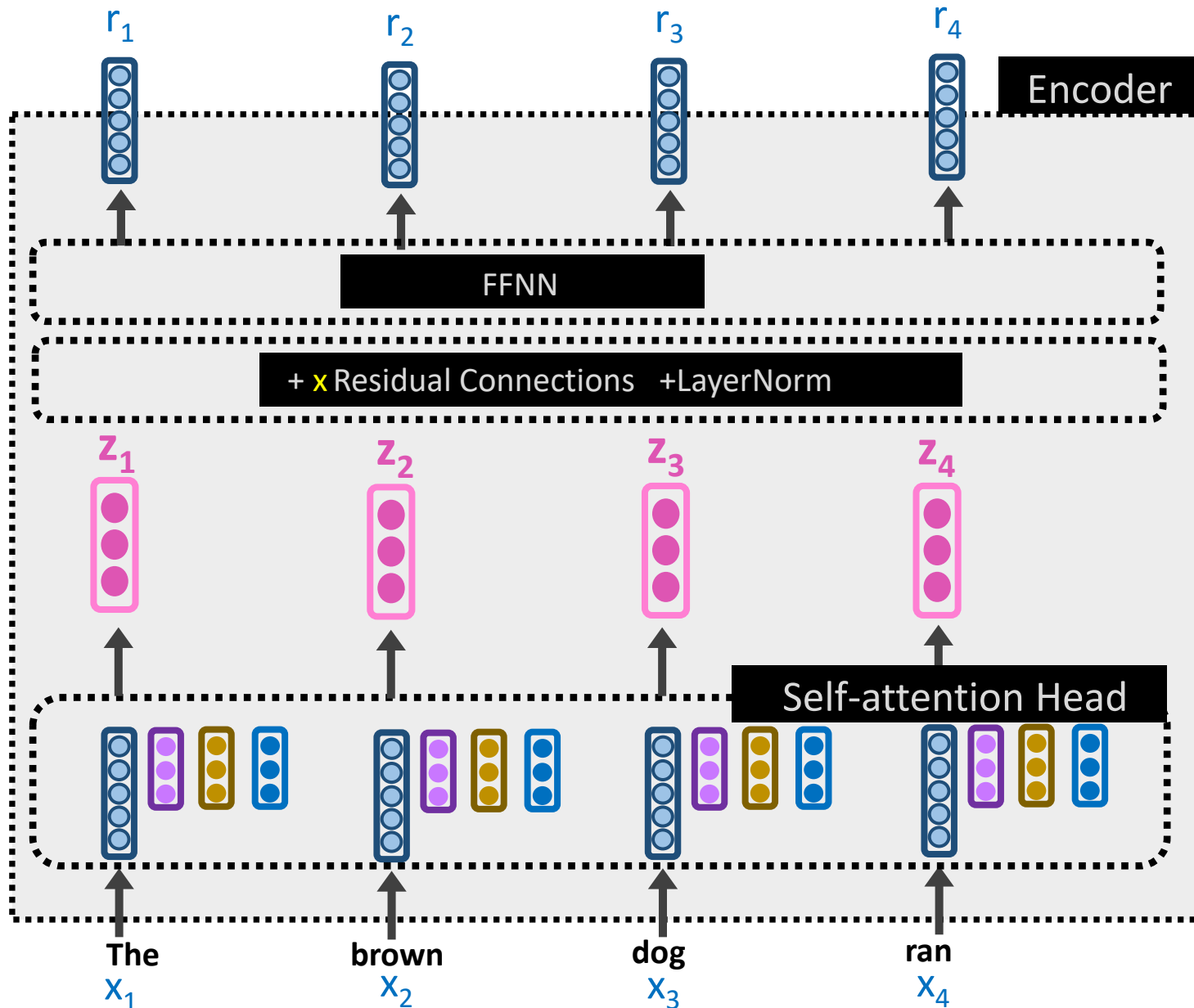
We concat w/ a **residual connection** to help ensure relevant info is getting forward passed.

We perform **LayerNorm** to stabilize the network and allow for proper gradient flow. You should do this after the FFNN, too.

Each  $z_i$  can be computed in **parallel**, unlike RNNs!

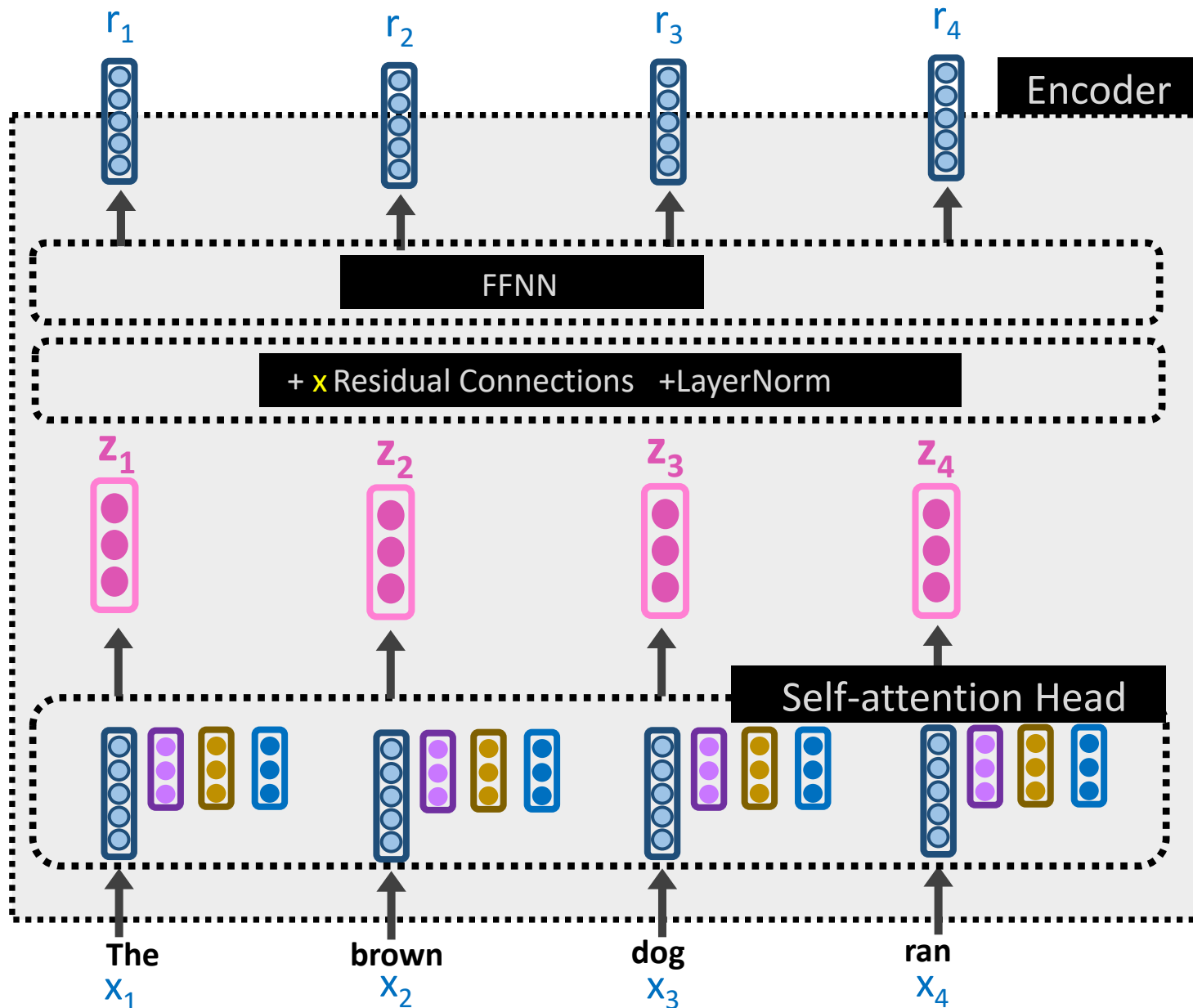


# Transformer Encoder



Yay! Our  $r_i$  vectors are our new representations, and this entire process is called a **Transformer Encoder**

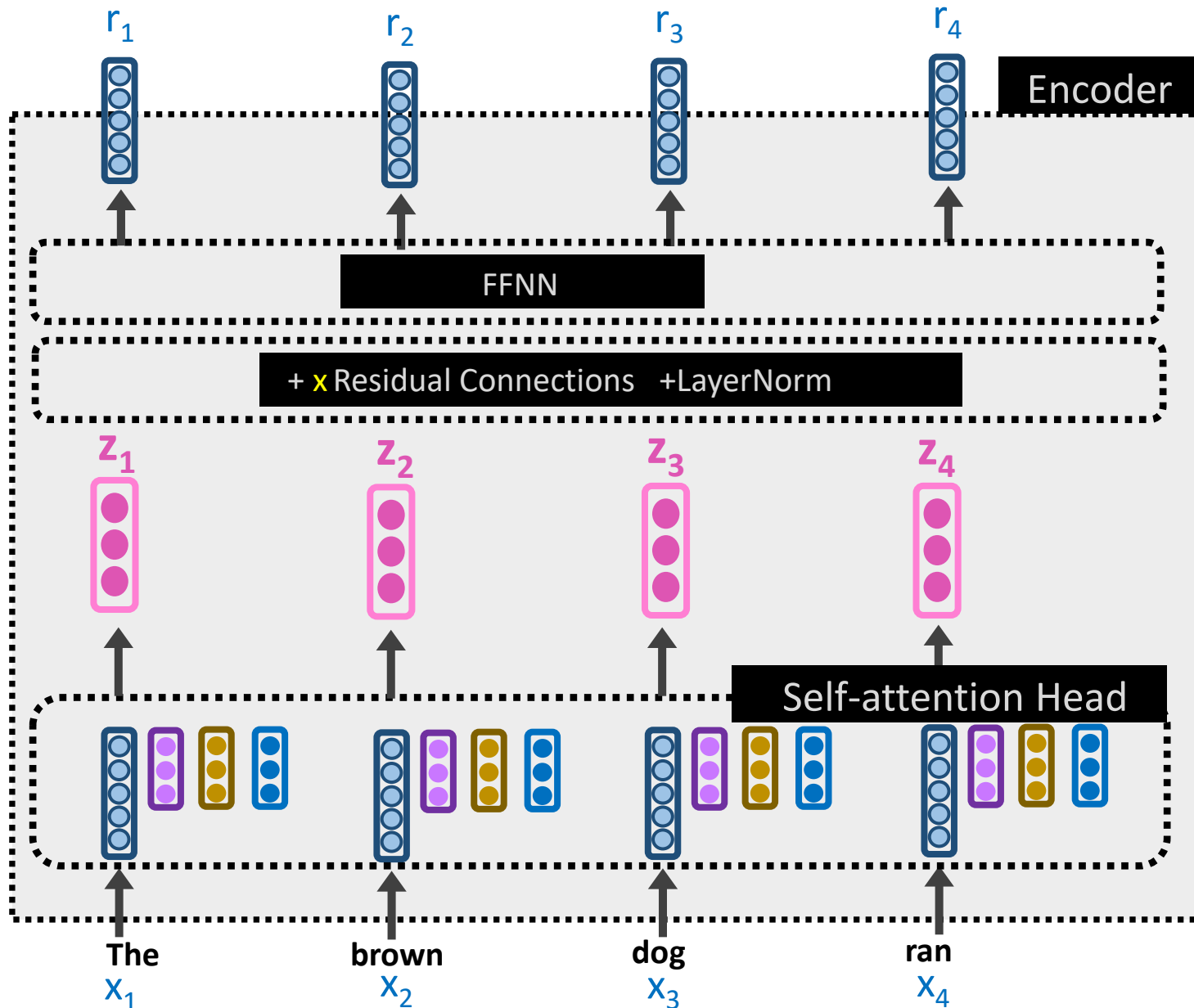
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**Problem:** there is no concept of positionality. Words are weighted as if a “bag of words”

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**Problem:** there is no concept of positionality. Words are weighted as if a “bag of words”

**Solution:** add to each input word  $x_i$  a **positional encoding**

Input to the model is now  $x_i + pos_i$

# How to encode position information?

- Self attention doesn't have a way to know whether an input token comes before or after another
  - Position is important in sequence modeling in NLP
- A way to introduce position information is add individual position encodings to the input for each position in the sequence

$$x_i = x_i + pos_i$$

Where  $pos_t$  is a position vector

# Properties of a good positional embedding

- It should output a unique encoding for each time-step (word's position in a sentence)
- Distance between any two time-steps should be consistent across sentences with different lengths.
  - The cat sat on the mat
  - The happy cat sat on the mat
- Our model should generalize to longer sentences without any efforts. Its values should be bounded.
- It must be deterministic.

# Absolute position embeddings

- Define a maximum context length you model can encode: say 1000 tokens.
  - Create a separate embedding table for each position.
  - Each index 1, 2, 3, ... gets an embedding.
  - Learn the embeddings with the model.
- Issues with Learned positions embeddings:
  - Maximum length that can be presented is limited (what if I get a 2000 token input)
  - Difficult to encode relative positions
    - The cat sat on the mat
    - The happy cat sat on the mat

# Functional (and fixed) position embeddings

## Sinusoidal embeddings

$$\vec{p}_t^{(i)} = f(t)^{(i)} := \begin{cases} \sin(\omega_k \cdot t), & \text{if } i = 2k \\ \cos(\omega_k \cdot t), & \text{if } i = 2k + 1 \end{cases}$$

where

$$\omega_k = \frac{1}{10000^{2k/d}}$$

$$\vec{p}_t = \begin{bmatrix} \sin(\omega_1 \cdot t) \\ \cos(\omega_1 \cdot t) \\ \\ \sin(\omega_2 \cdot t) \\ \cos(\omega_2 \cdot t) \\ \\ \vdots \\ \\ \sin(\omega_{d/2} \cdot t) \\ \cos(\omega_{d/2} \cdot t) \end{bmatrix}$$

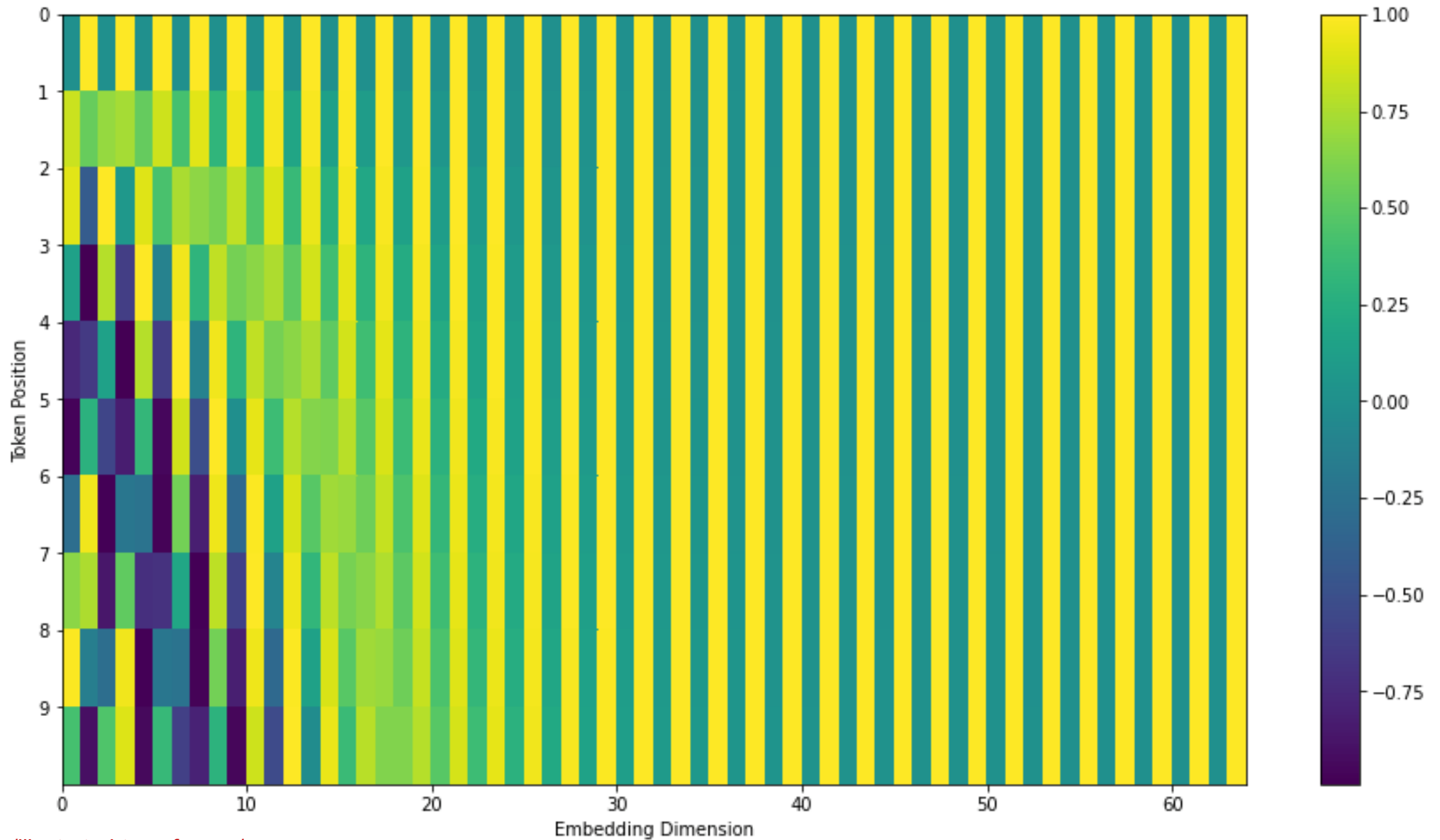
The frequencies are decreasing along the vector dimension. It forms a geometric progression on the wavelengths.

# Sinusoidal Embeddings: Intuition

0 :	0	0	0	0	8 :	1	0	0	0
1 :	0	0	0	1	9 :	1	0	0	1
2 :	0	0	1	0	10 :	1	0	1	0
3 :	0	0	1	1	11 :	1	0	1	1
4 :	0	1	0	0	12 :	1	1	0	0
5 :	0	1	0	1	13 :	1	1	0	1
6 :	0	1	1	0	14 :	1	1	1	0
7 :	0	1	1	1	15 :	1	1	1	1



# Position Encodings



# Variants of Positional Embeddings

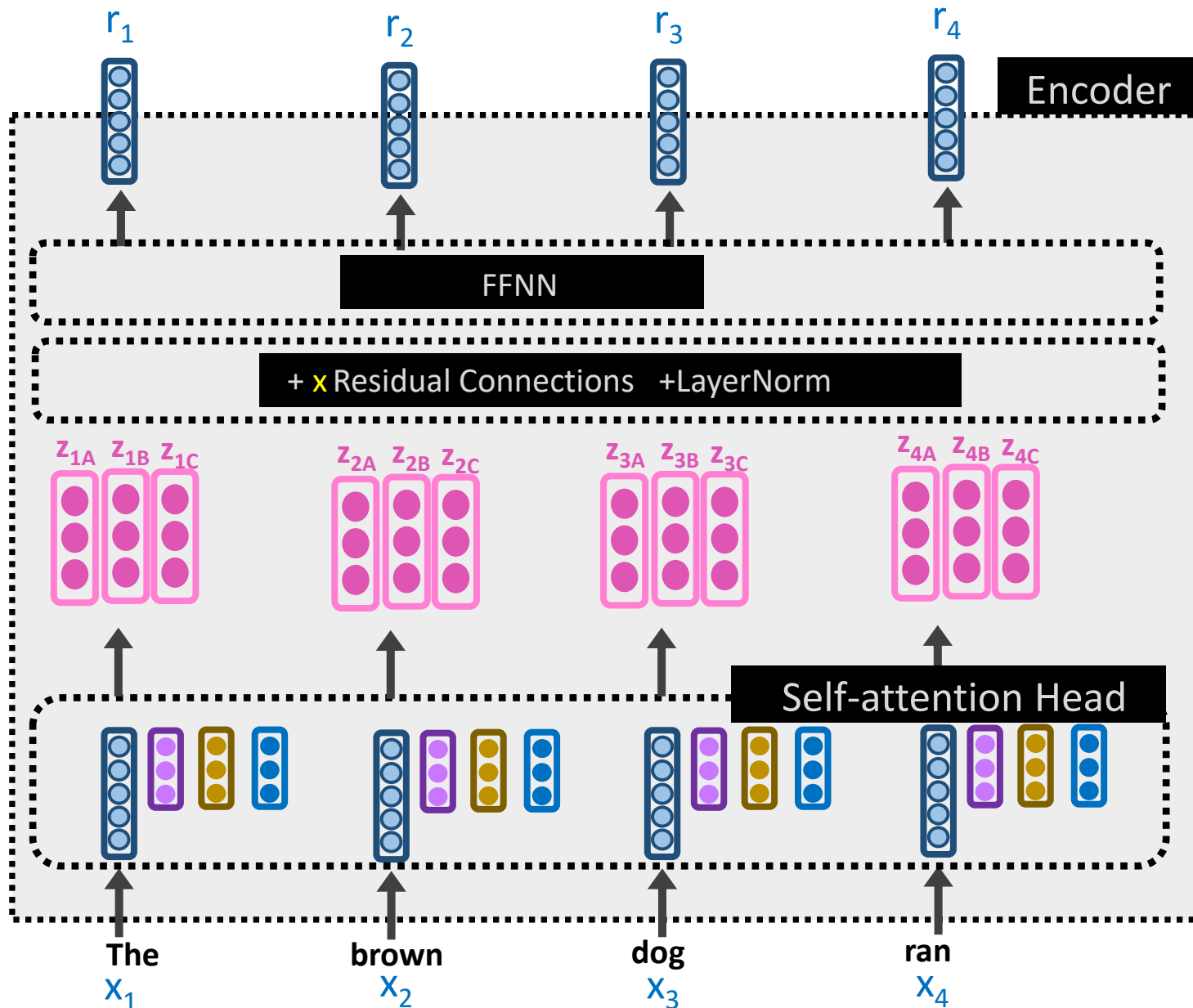
- Rotary Positional Embeddings (RoPE): [\[2104.09864\] RoFormer: Enhanced Transformer with Rotary Position Embedding \(arxiv.org\)](#)
- AliBi: [\[2108.12409\] Train Short, Test Long: Attention with Linear Biases Enables Input Length Extrapolation \(arxiv.org\)](#)
- No embeddings(!?): [\[2203.16634\] Transformer Language Models without Positional Encodings Still Learn Positional Information \(arxiv.org\)](#)

A **Self-Attention Head** has just one set of query/key/value weight matrices  $\mathbf{w}_q, \mathbf{w}_k, \mathbf{w}_v$

Words can relate in many ways, so it's restrictive to rely on just one Self-Attention Head in the system.

Let's create Multi-headed Self-Attention

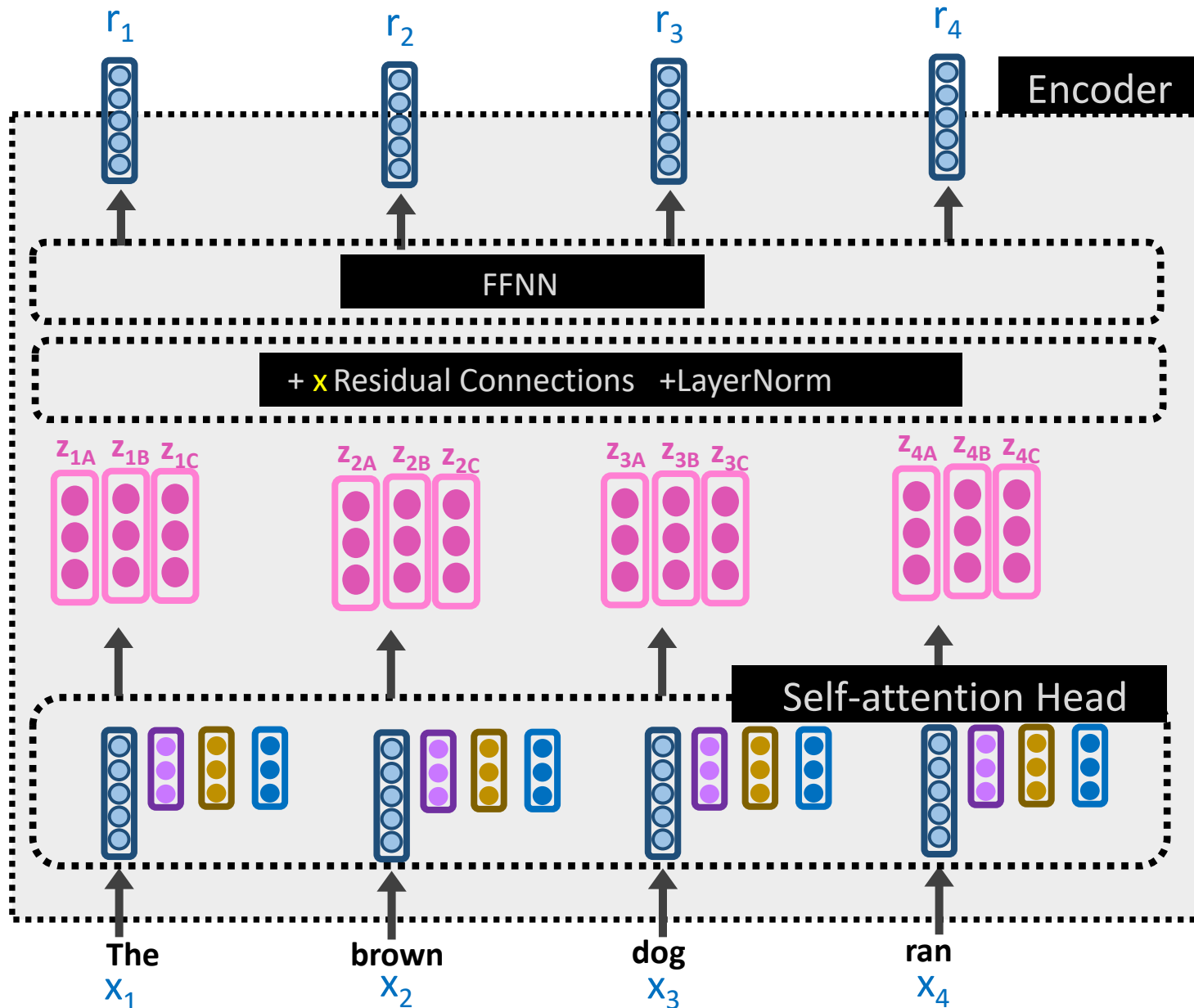
# Multi-head Attention



Each **Self-Attention Head** produces a  $z_i$  vector using query, key, and value vectors

We can, in parallel, use **multiple heads** and concat the  $z_i$ 's. For each input create multiple query, key, and value vectors

# Transformer Encoder

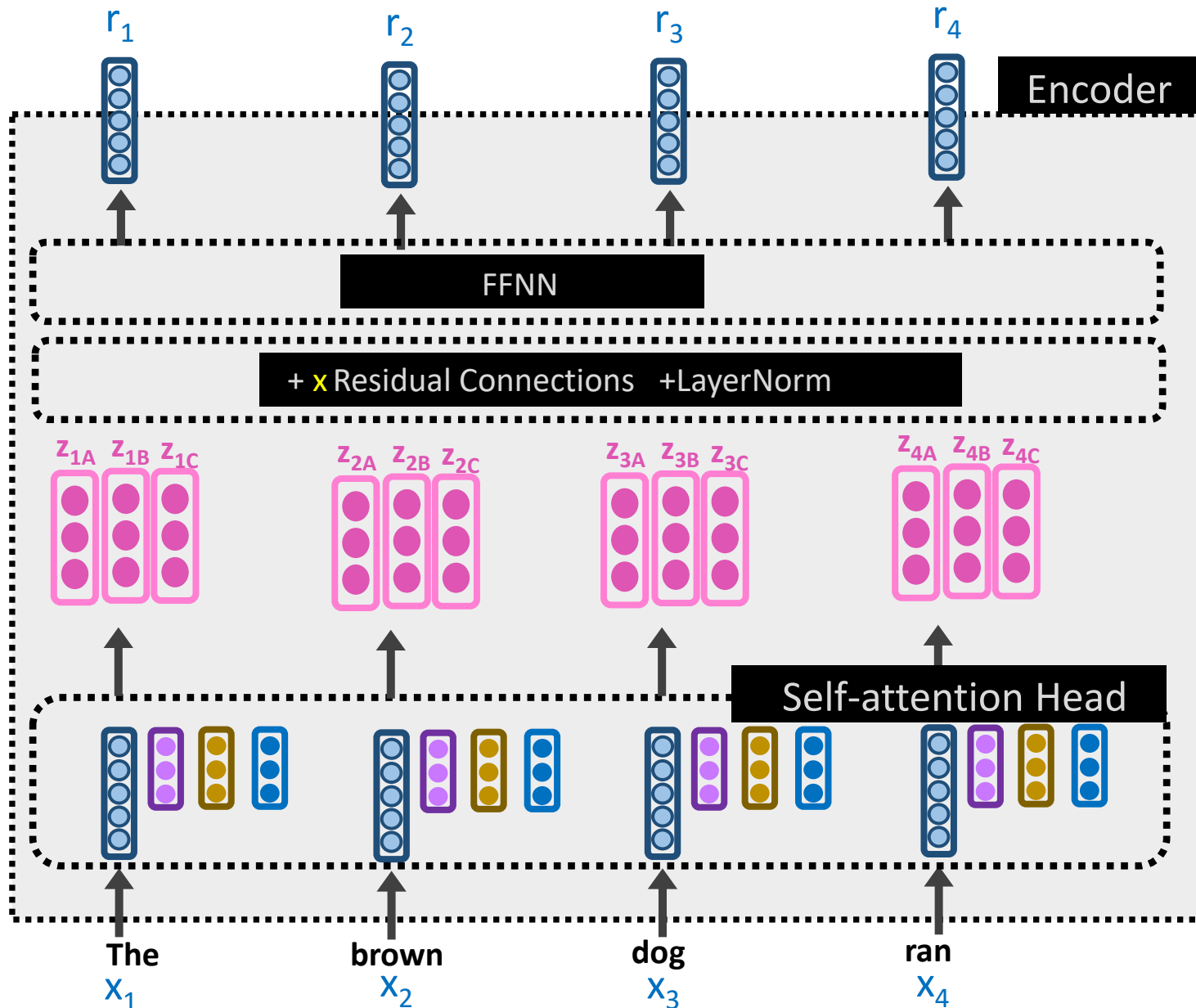


**To recap:** all of this looks fancy, but ultimately it's just producing a very good

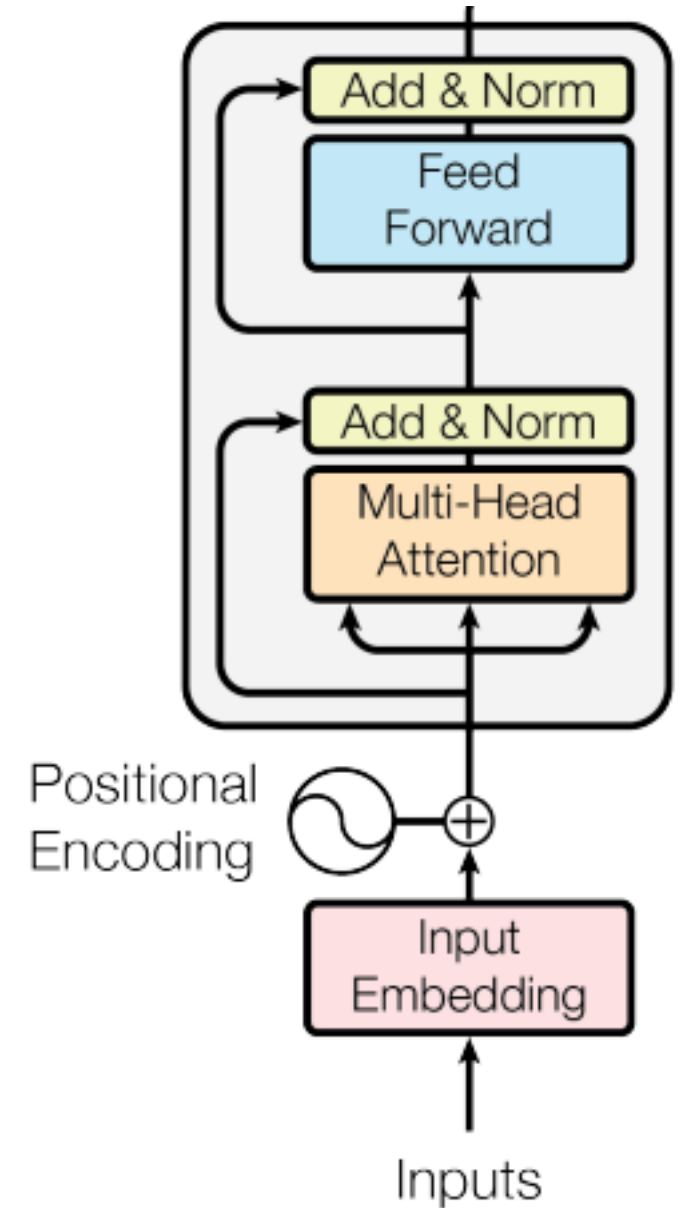
**contextualized embedding**

$r_i$  of each word  $x_i$

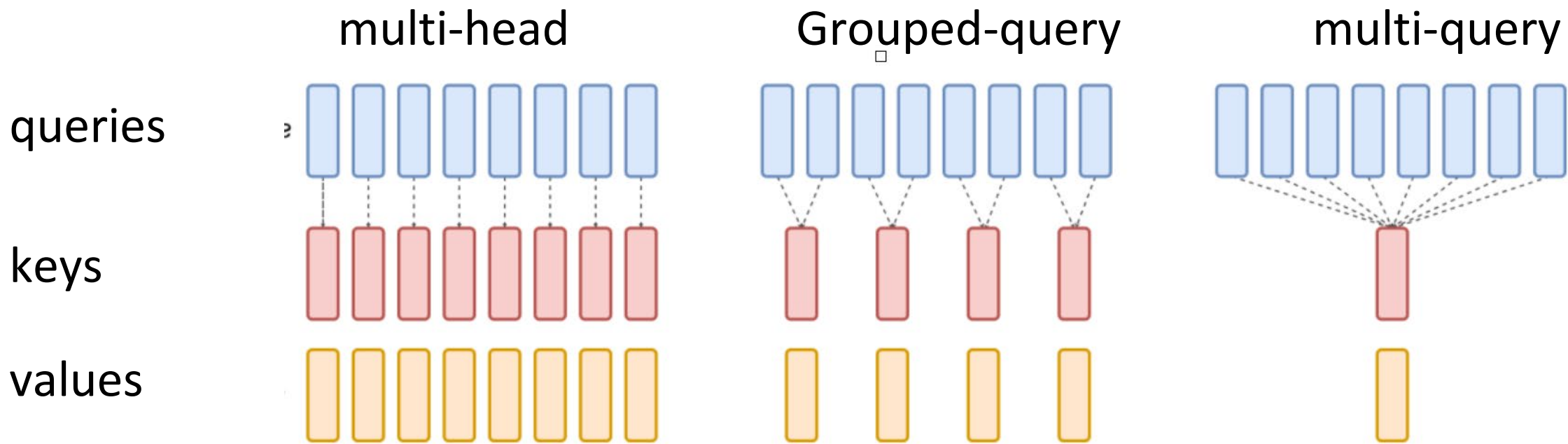
# Transformer Encoder



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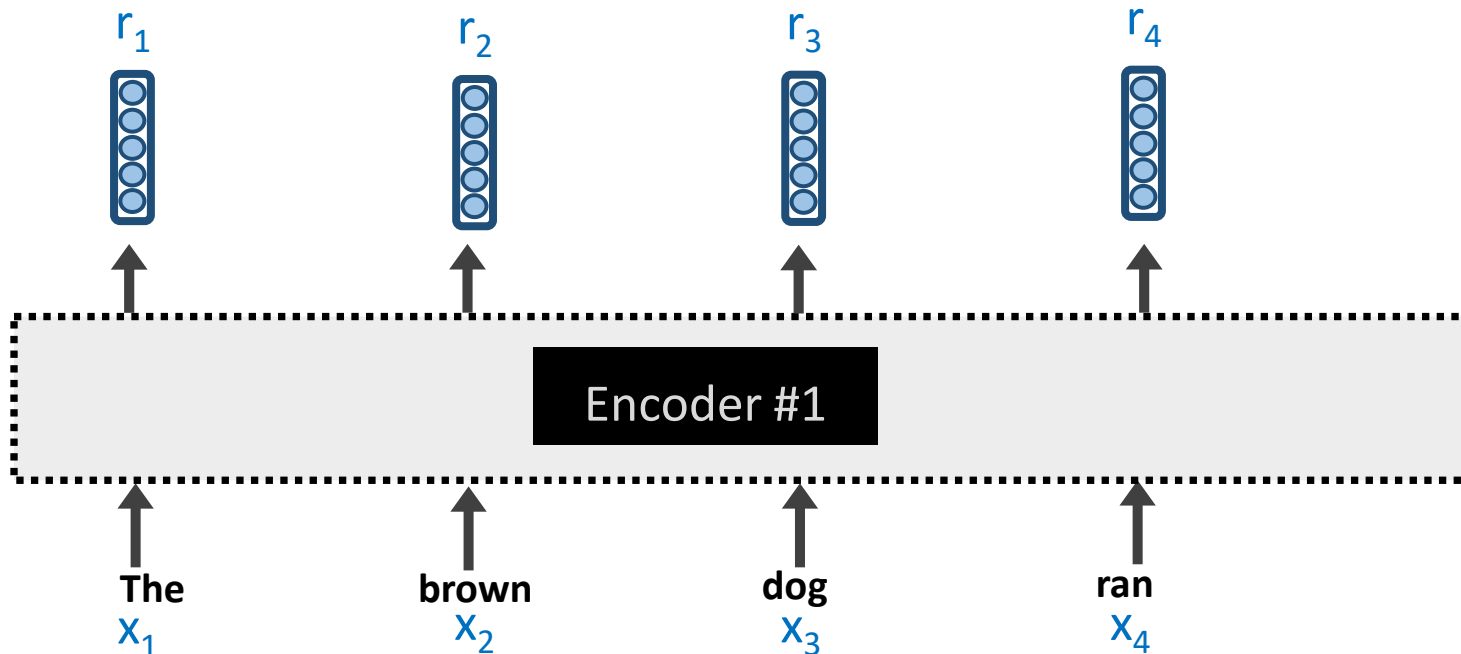


# Variants of multi-head attention attention



# Transformer Encoder

**To recap:** all of this looks fancy, but ultimately it's just producing a very good **contextualized embedding**  $r_i$  of each word  $x_i$



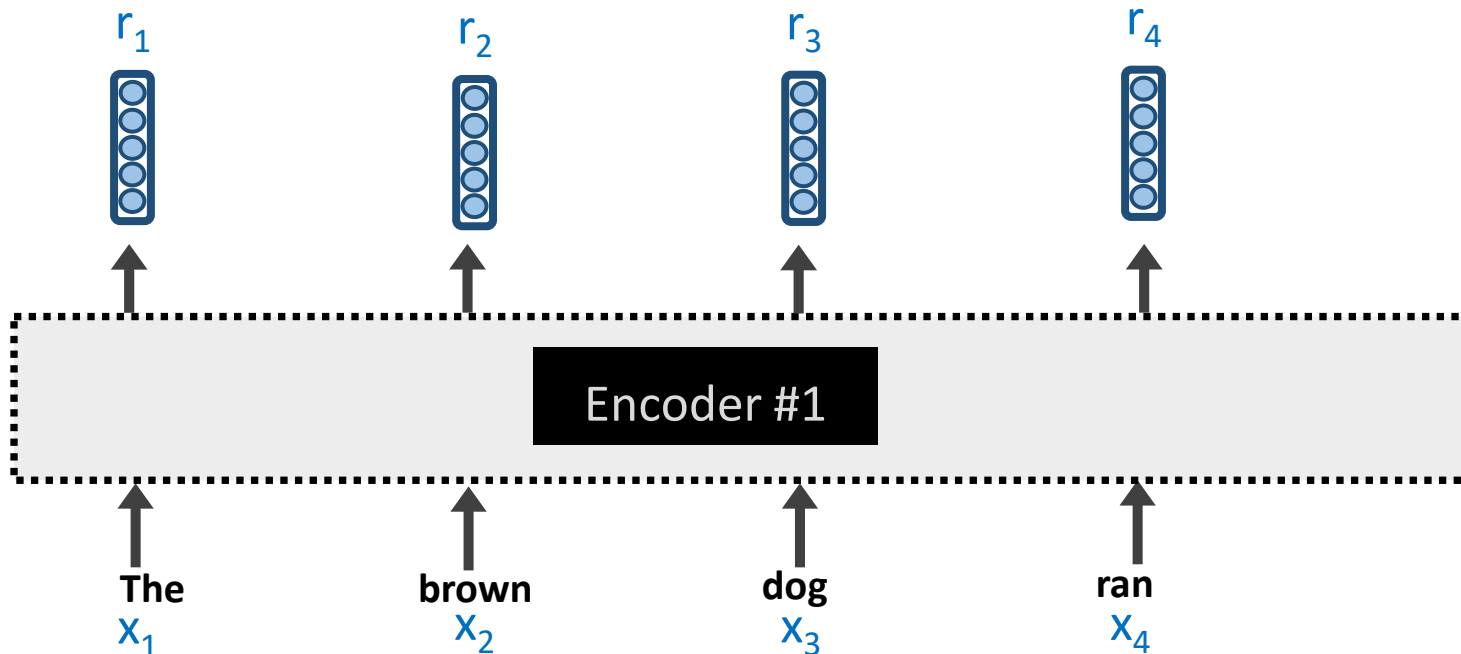


# Transformer Encoder

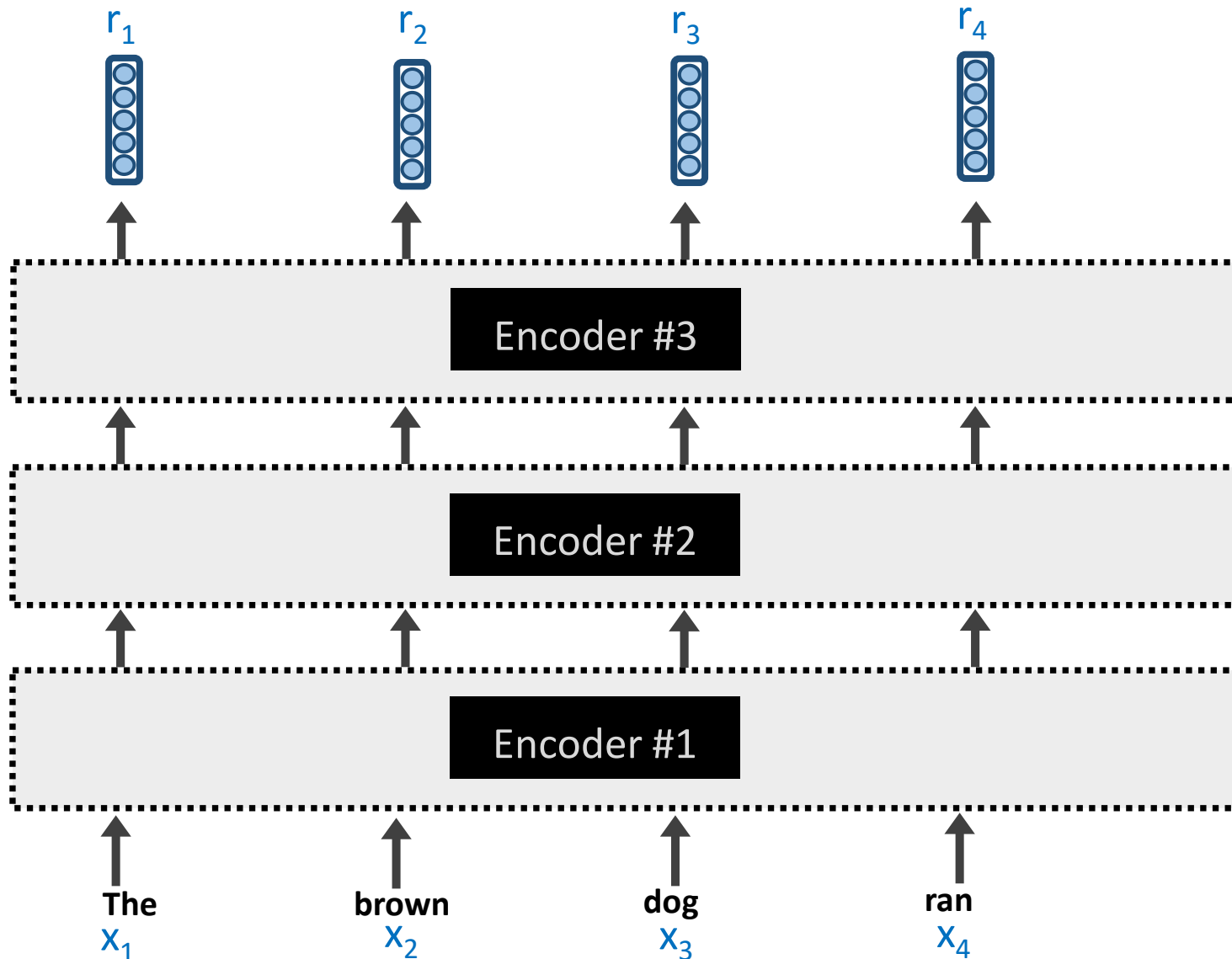
**To recap:** all of this looks fancy, but ultimately it's just producing a very good **contextualized embedding**

$r_i$  of each word  $x_i$

Why stop with just 1 **Transformer Encoder**? We could stack several!



# Transformer Encoder



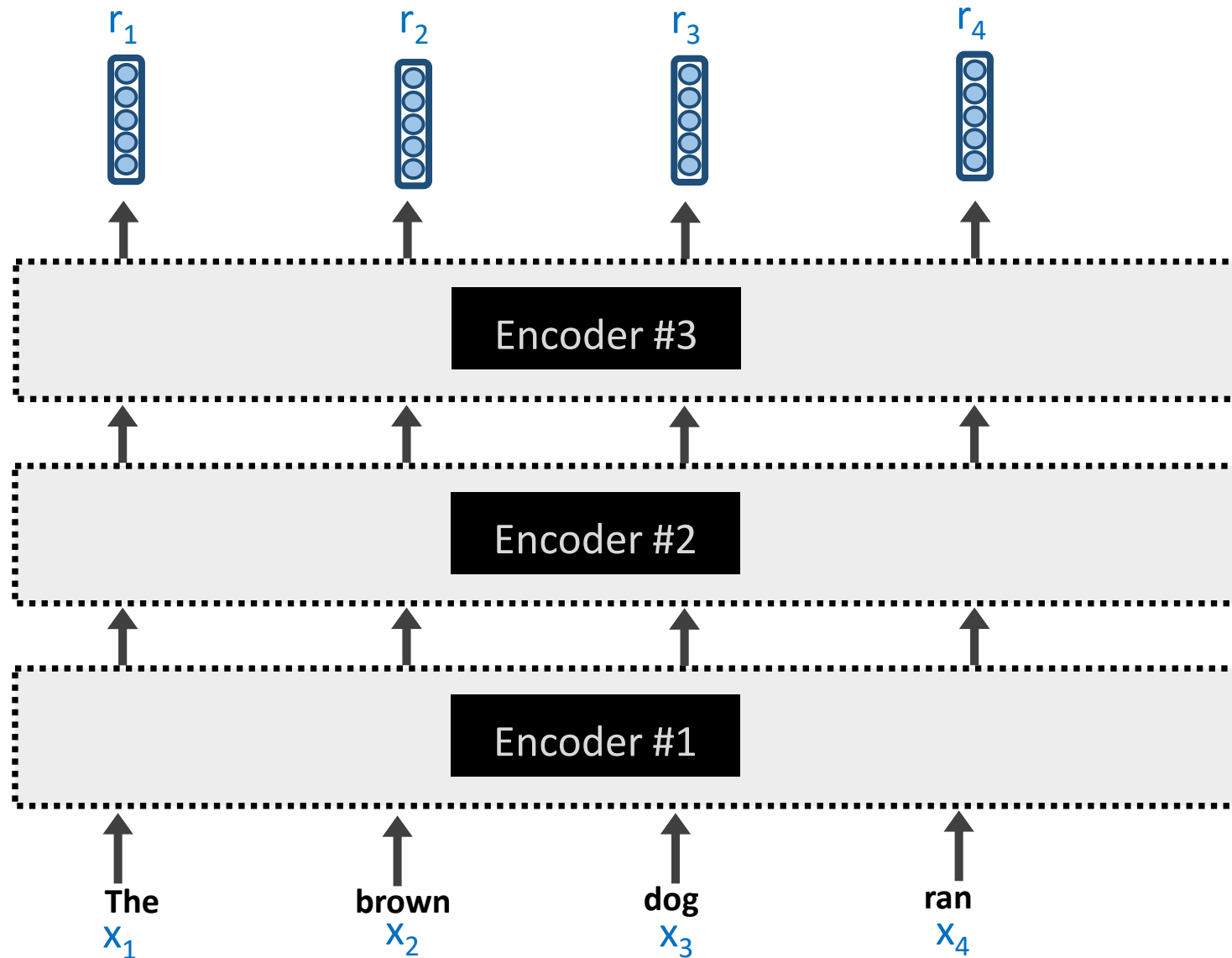
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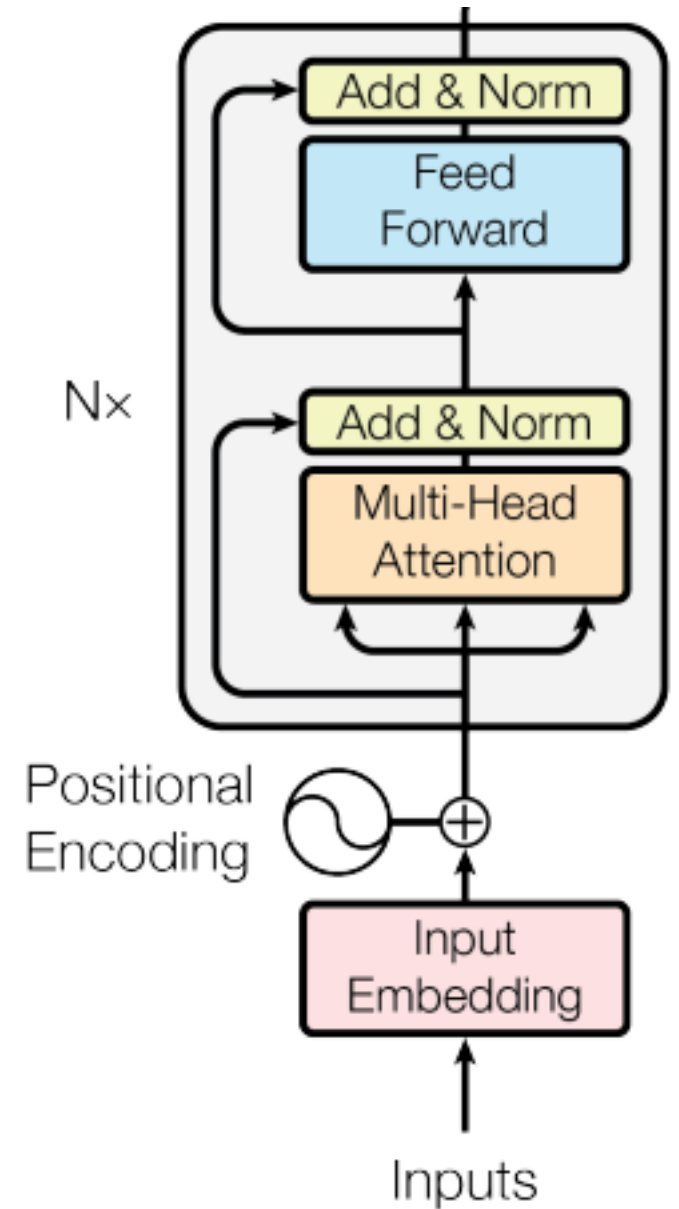
$r_i$  of each word  $x_i$

Why stop with just 1  
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# Transformer Encoder



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The original Transformer model was intended for Machine Translation, so it had **Decoders**, too

# Outline



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Language Modeling With Transformers

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Self-Attention



Transformer Encoder



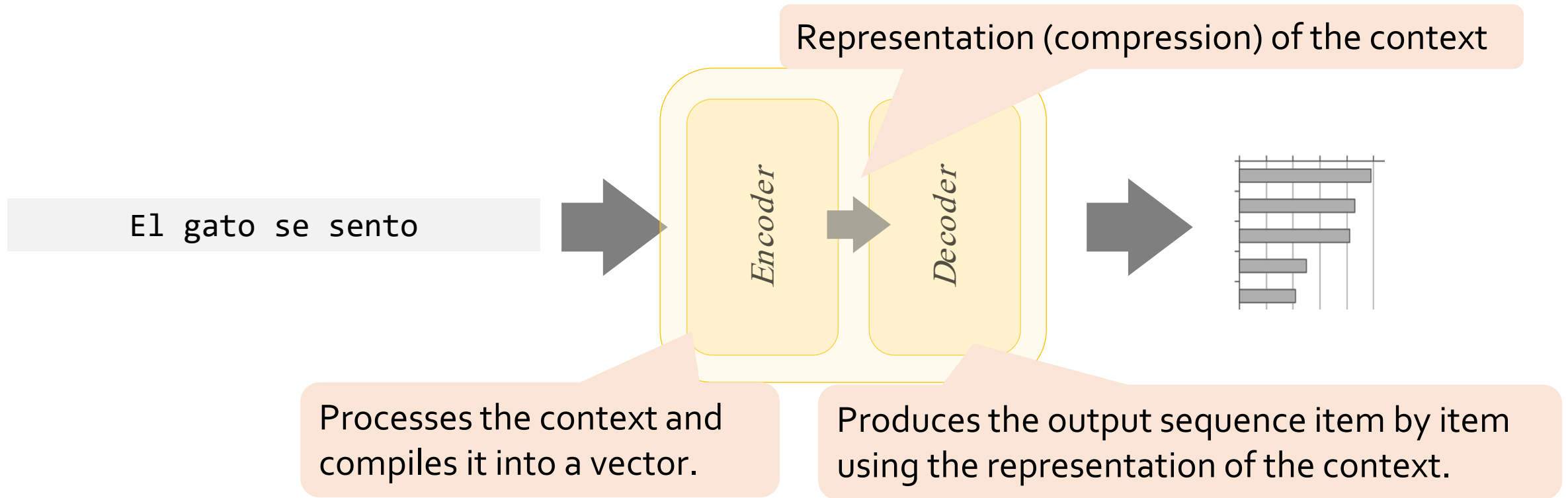
Transformer Decoder



Language Modeling With Transformers

# Encoder-Decoder Architectures

- Original transformer had two sub-models.

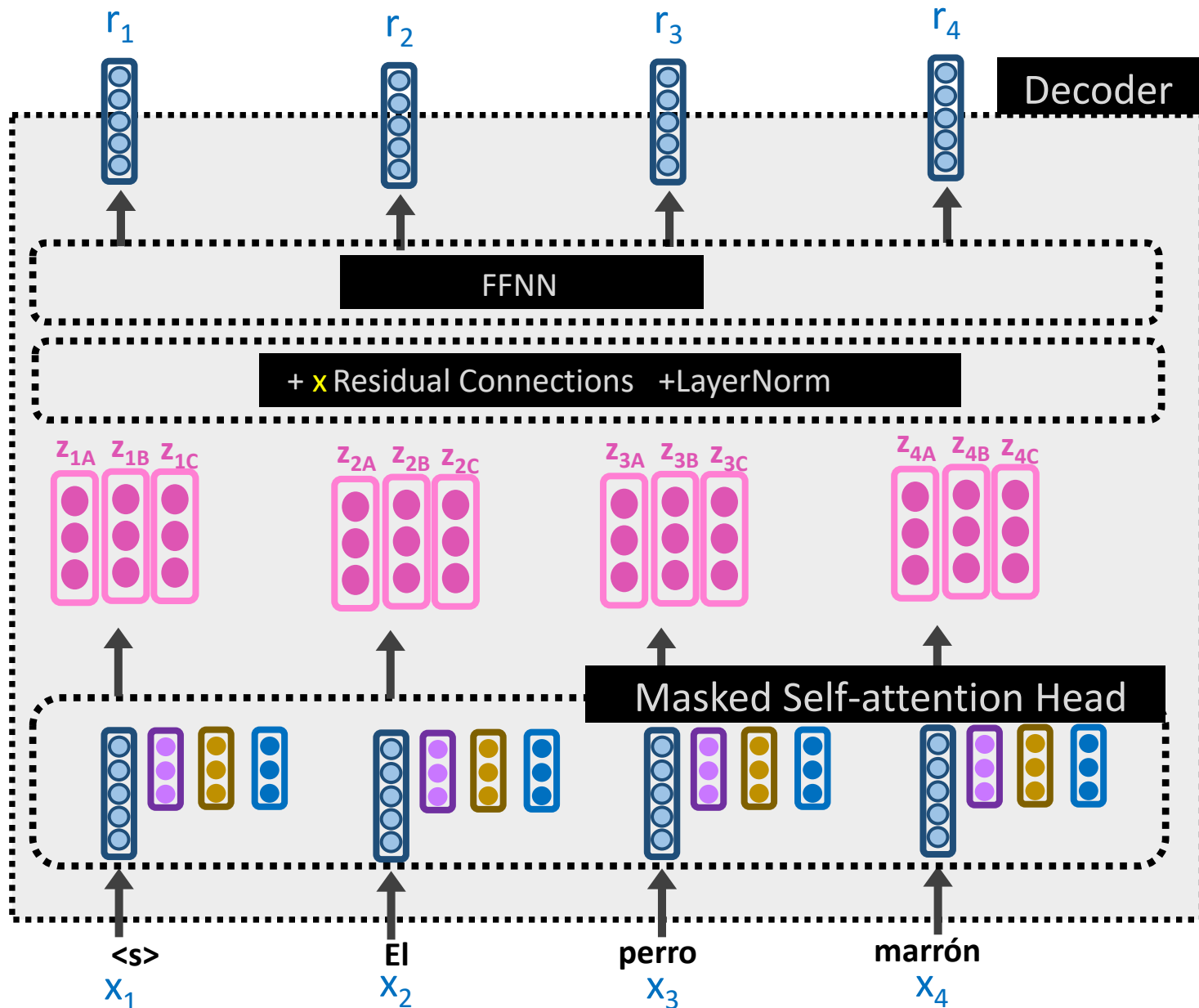


# Encoder-Decoder Architectures

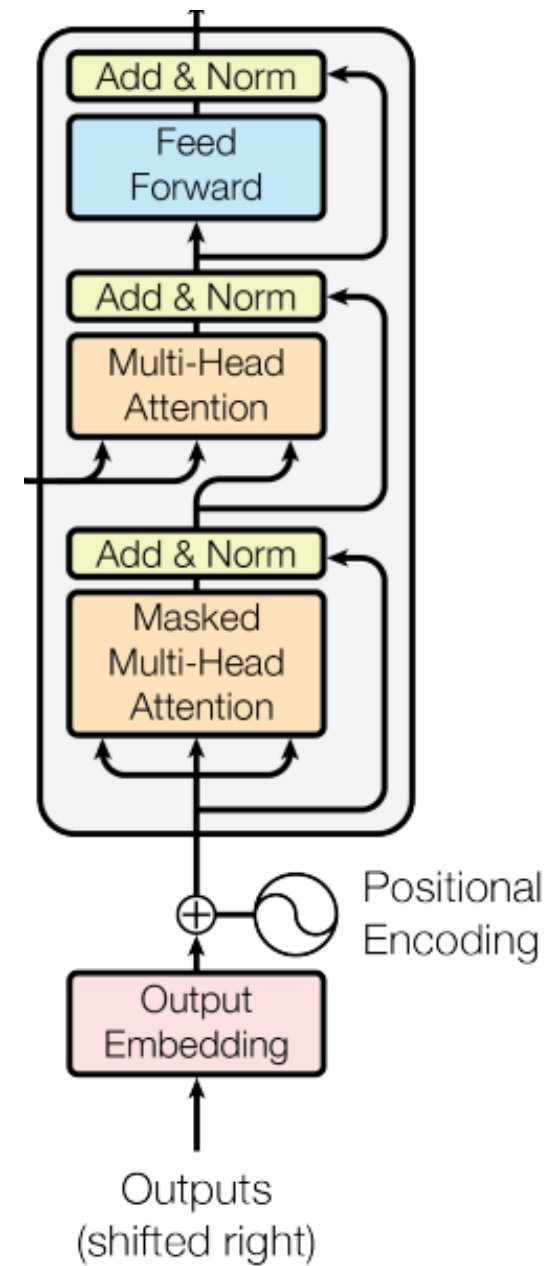




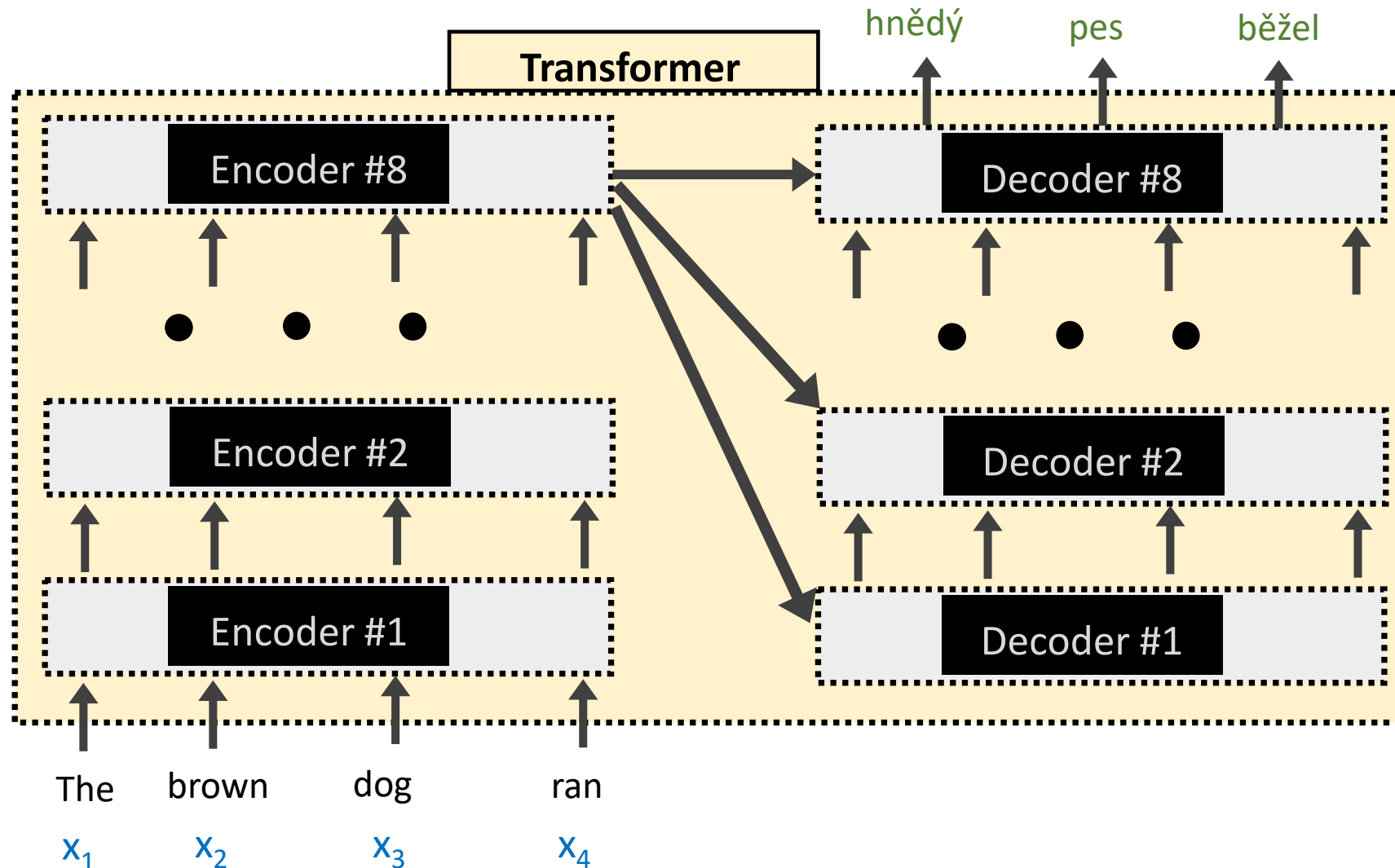
# Transformer Decoder



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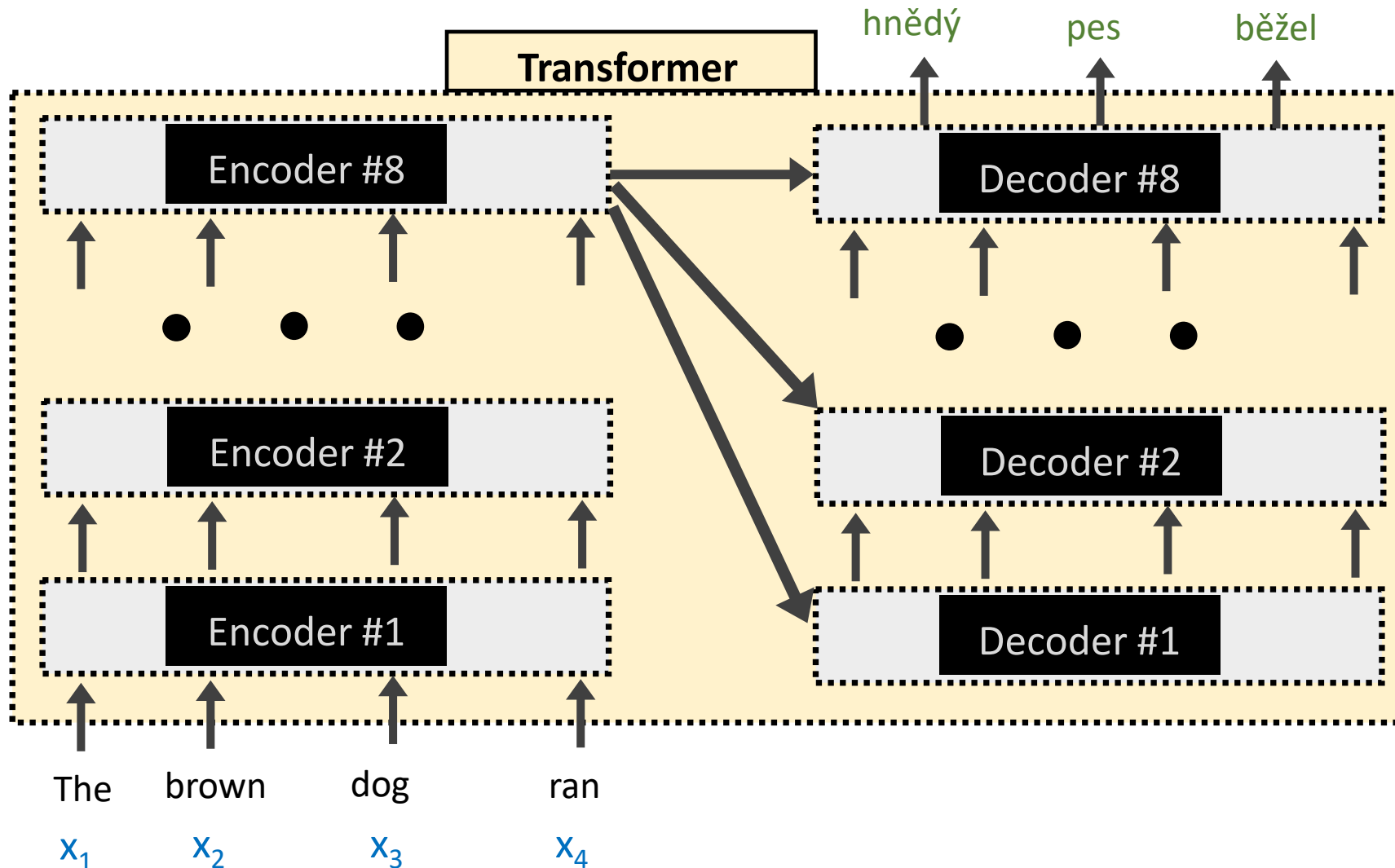
# Transformer Encoders and Decoders



**Transformer Encoders**  
produce **contextualized embeddings** of each word

**Transformer Decoders**  
generate new sequences of text

# Transformer Encoders and Decoders

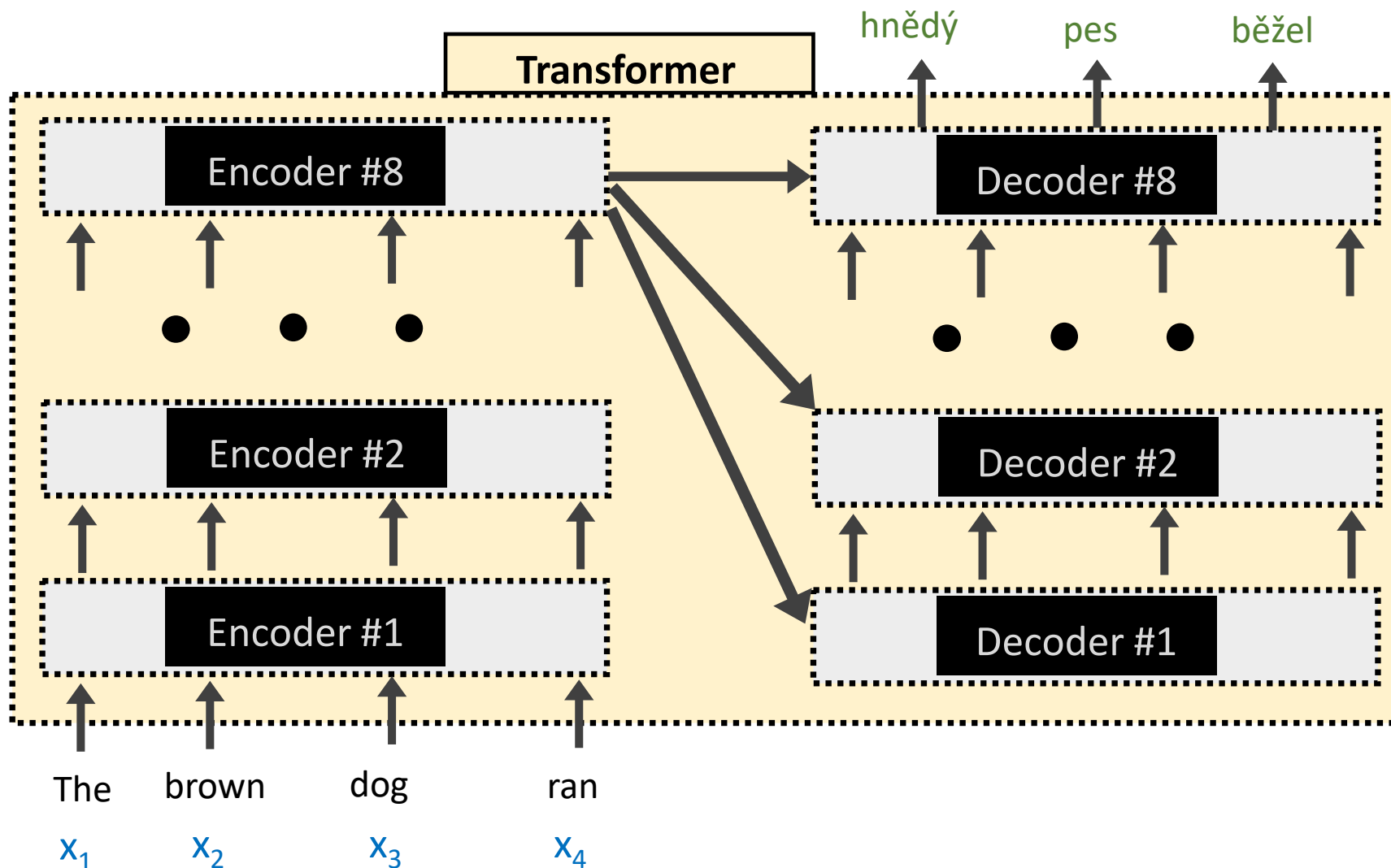


## NOTE

Transformer Decoders are identical to the Encoders, except they have an additional **Attention Head** in between the Self-Attention and FFNN layers.

This additional **Attention Head** focuses on parts of the encoder's representations.

# Transformer Encoders and Decoders

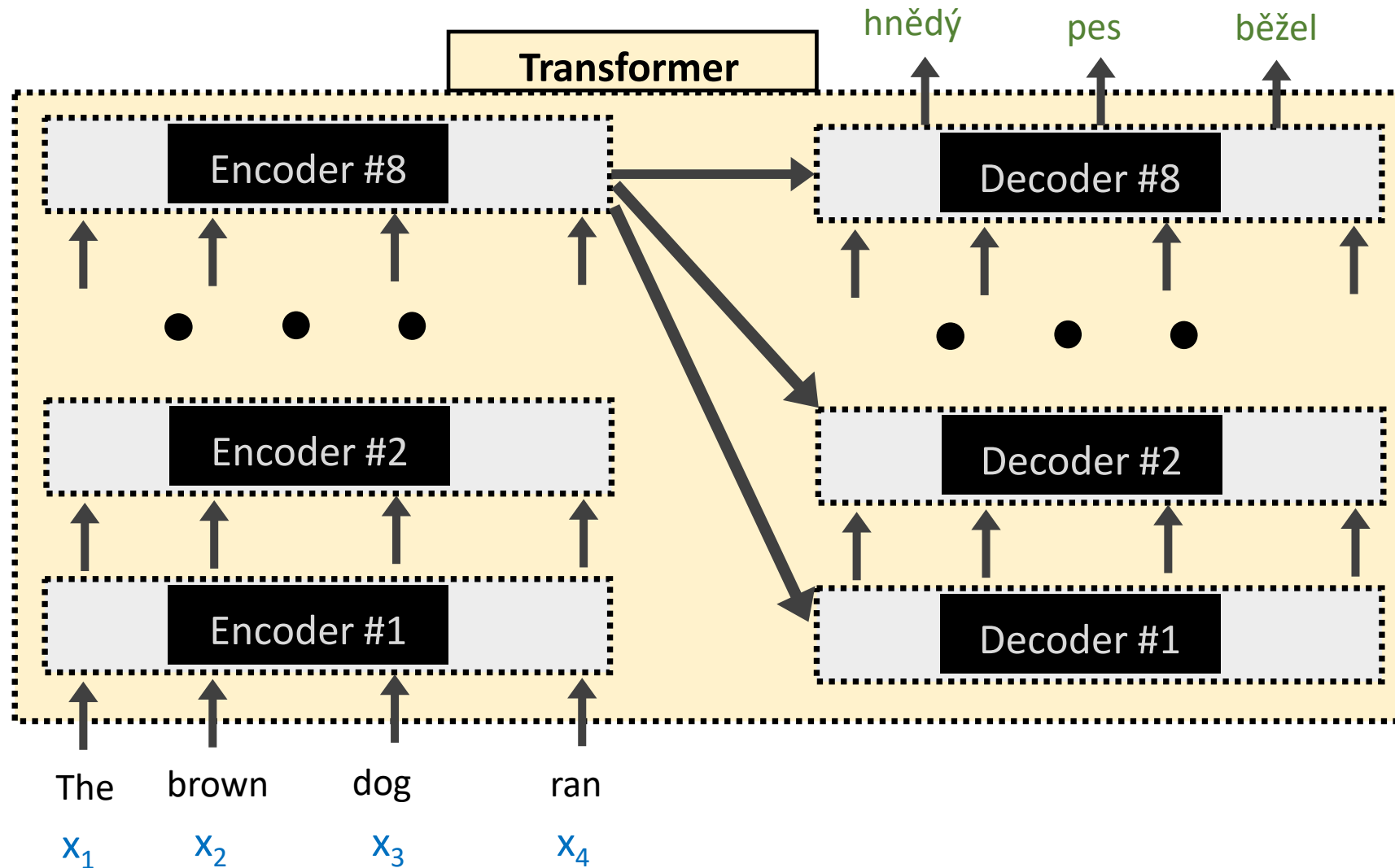


## NOTE

The **query** vector for a Transformer **Decoder's Attention Head** (not Self-Attention Head) is from the output of the previous decoder layer.

However, the **key** and **value** vectors are from the **Transformer Encoders'** outputs.

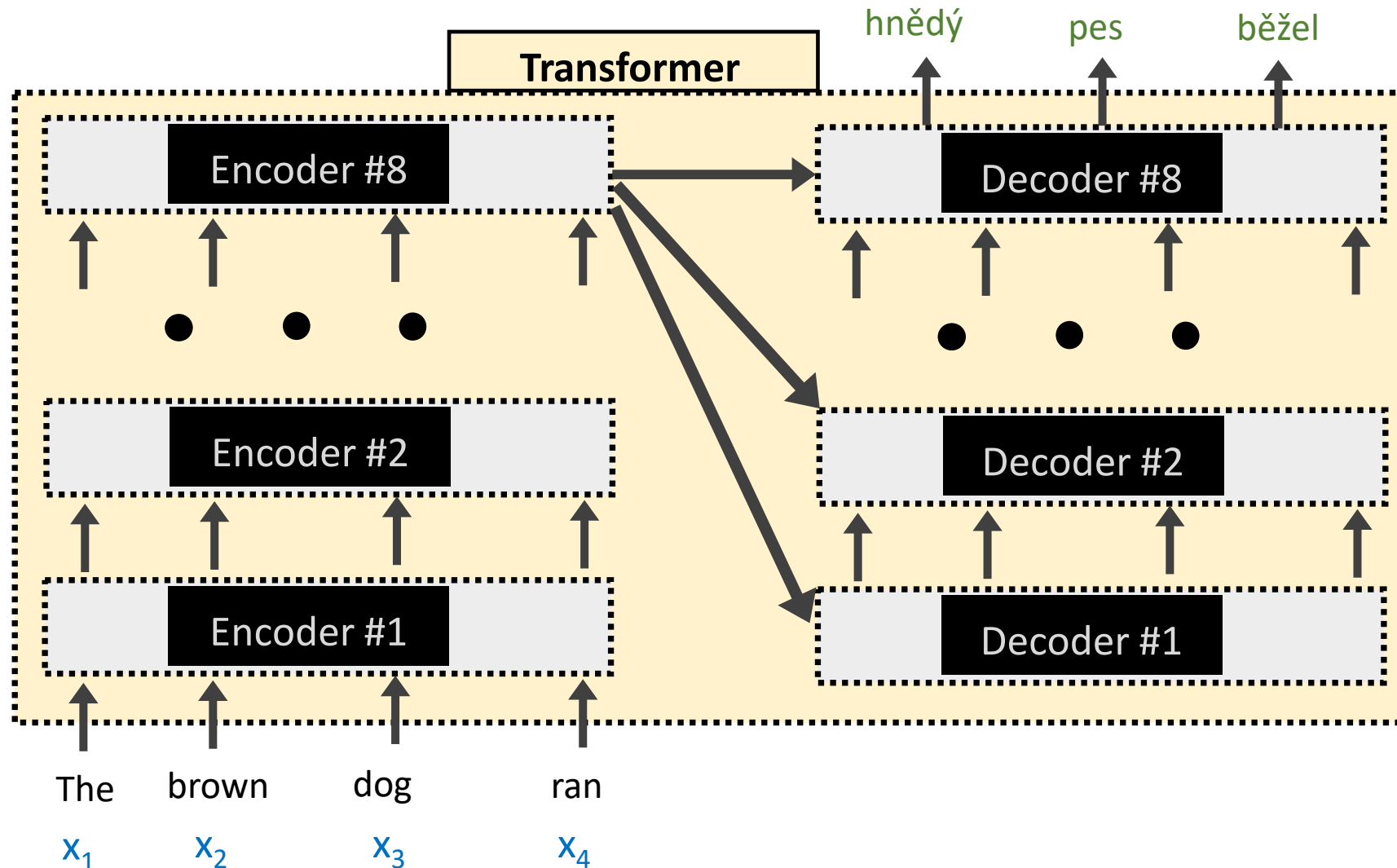
# Transformer Encoders and Decoders



## NOTE

The **query**, **key**, and **value** vectors for a Transformer **Decoder's Self-Attention Head** (not Attention Head) are all from the output of the previous decoder layer.

# Transformer Encoders and Decoders



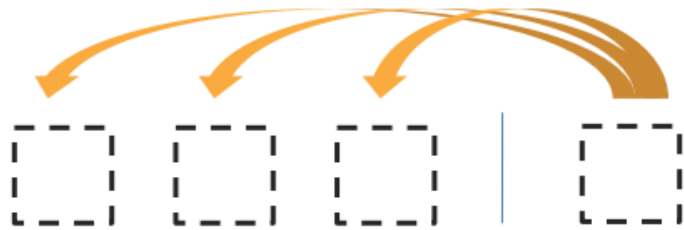
## IMPORTANT

The Transformer **Decoders** have **positional embeddings**, too, just like the **Encoders**.

Critically, each position is **only allowed to attend to the previous indices**. This *masked* Attention preserves it as being an auto-regressive LM.

# Transformer [Vaswani et al. 2017]

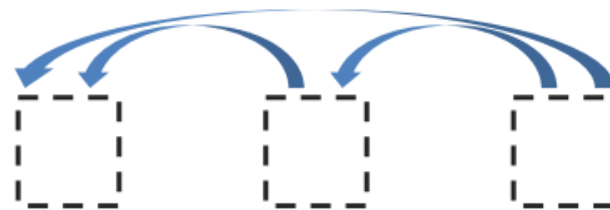
- An **encoder-decoder** architecture built with **attention** modules.
- 3 forms of attention



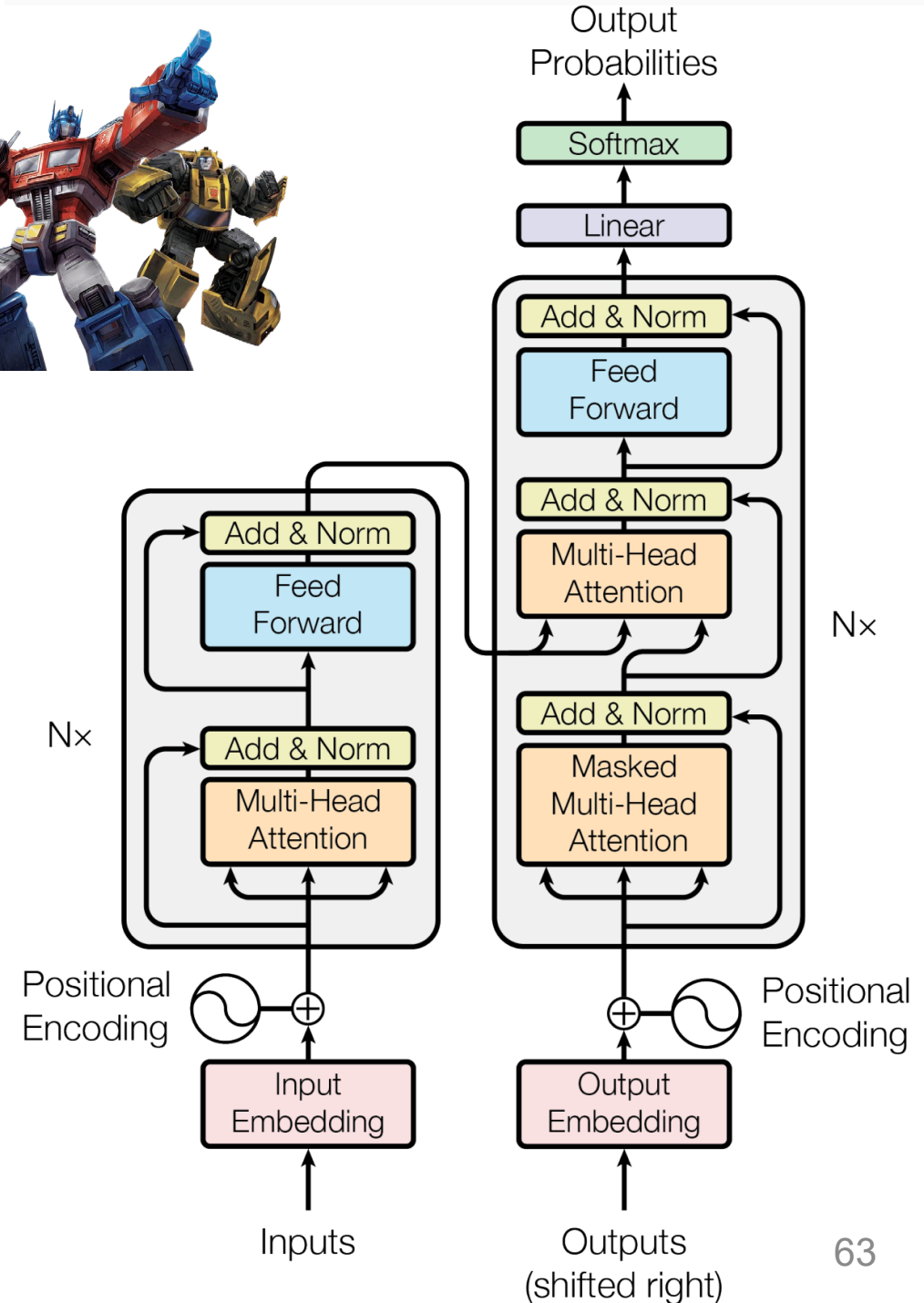
Encoder-Decoder Attention



Encoder Self-Attention

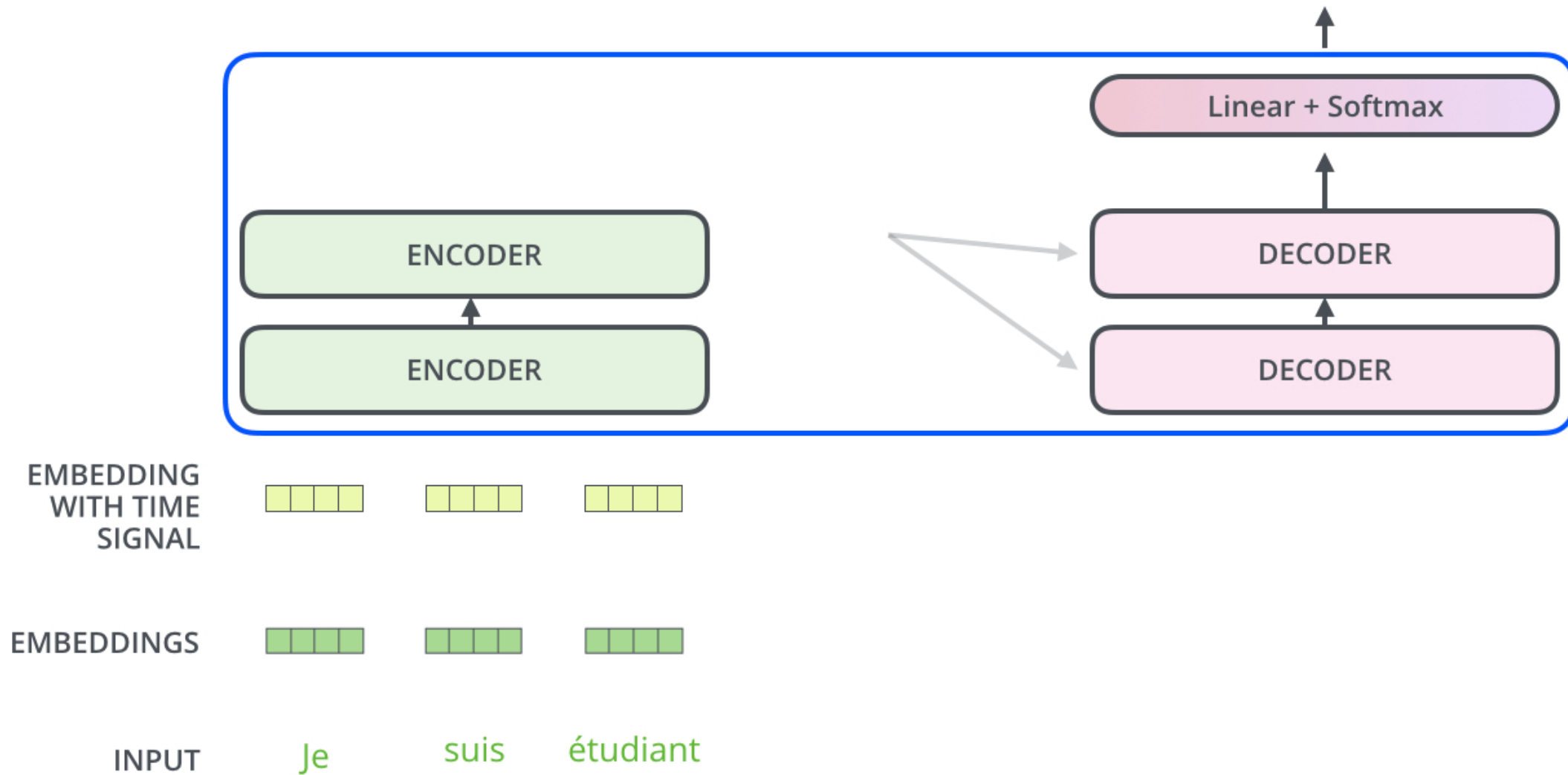


Masked Decoder Self-Attention



Decoding time step: 1 2 3 4 5 6

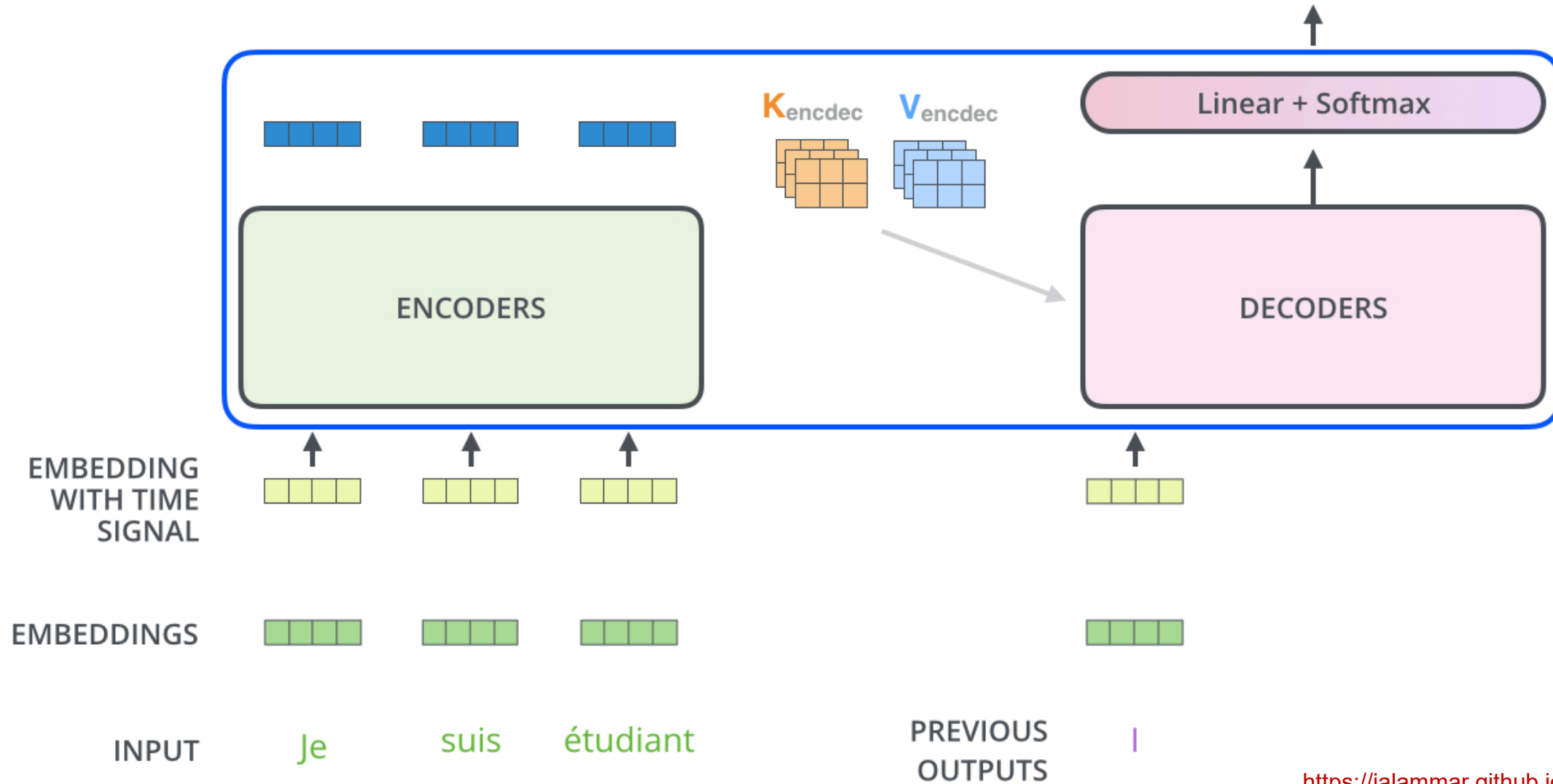
OUTPUT





Decoding time step: 1 2 3 4 5 6

OUTPUT |



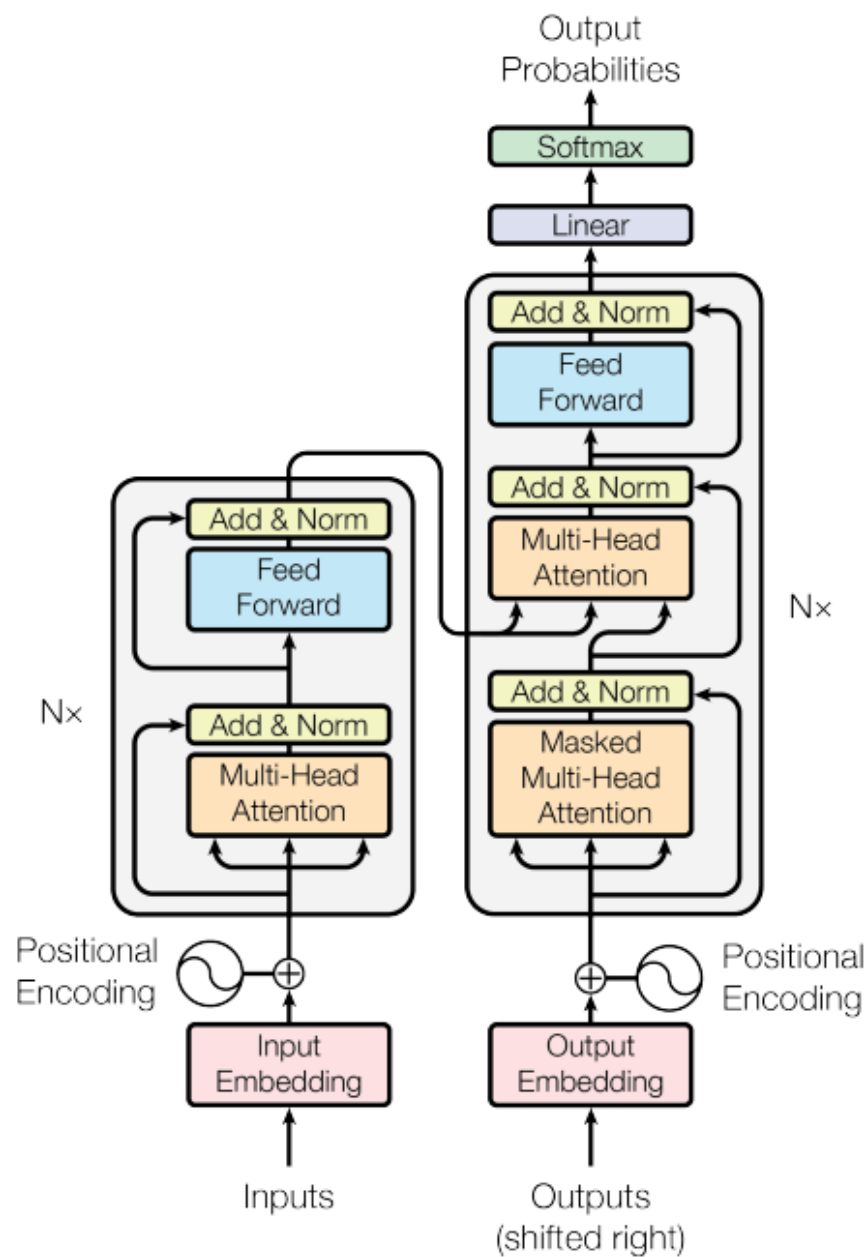


Figure 1: The Transformer - model architecture.

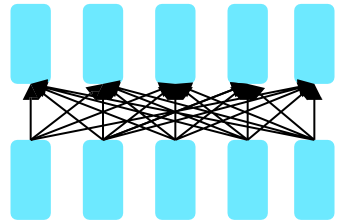
# Impact of Transformers

- Let to better predictive models of language ala GPTs!

Model	Layers	Heads	Perplexity
LSTMs ( <a href="#">Grave et al., 2016</a> )	-	-	40.8
QRNNs ( <a href="#">Merity et al., 2018</a> )	-	-	33.0
Transformer	16	16	19.8

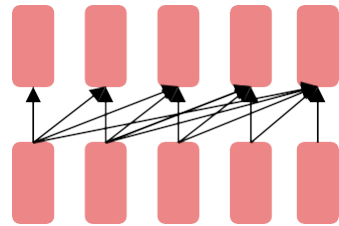
# Impact of Transformers

- A building block for a variety of LMs



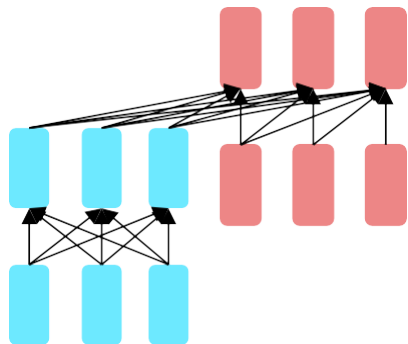
**Encoders**

- ❖ **Examples:** BERT, RoBERTa, SciBERT.
- ❖ Captures bidirectional context. How do we pretrain them?



**Decoders**

- ❖ **Examples:** GPT-2, GPT-3, Llama models, and many many more
- ❖ Other name: **causal or auto-regressive language model**
- ❖ Nice to generate from; can't condition on future words



**Encoder-  
Decoders**

- ❖ **Examples:** Transformer, T5, BART
- ❖ What's the best way to pretrain them?

# Transformer LMs + Scale = LLMs

- 2 main dimensions:
- Model size, pretraining data size

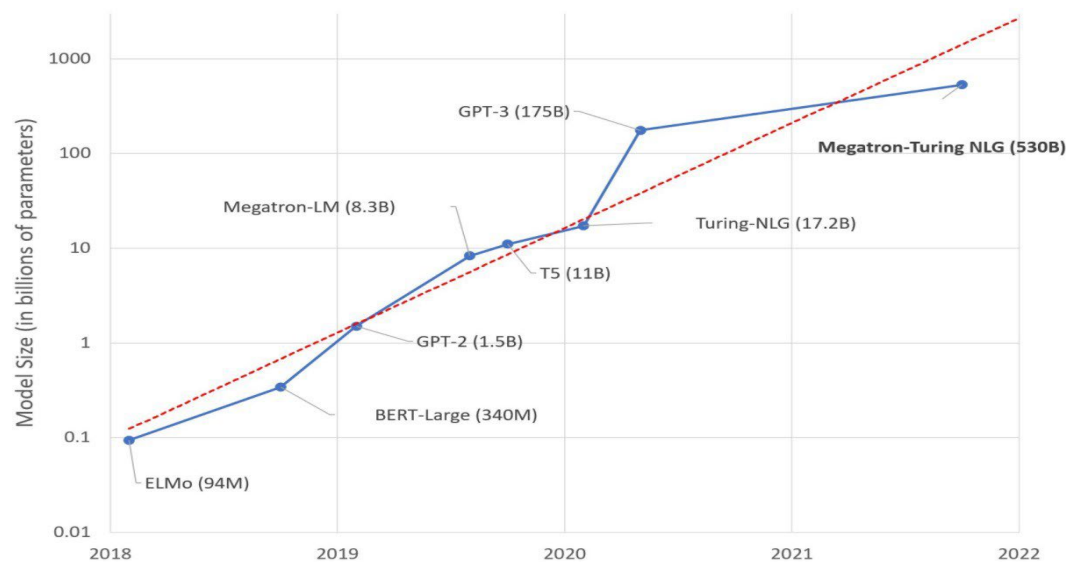
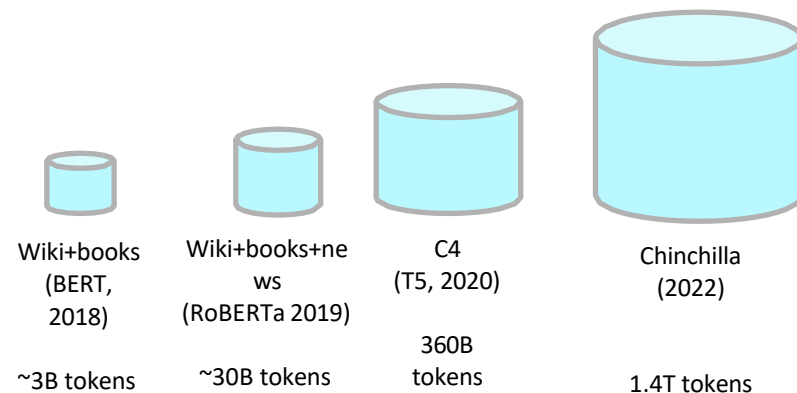
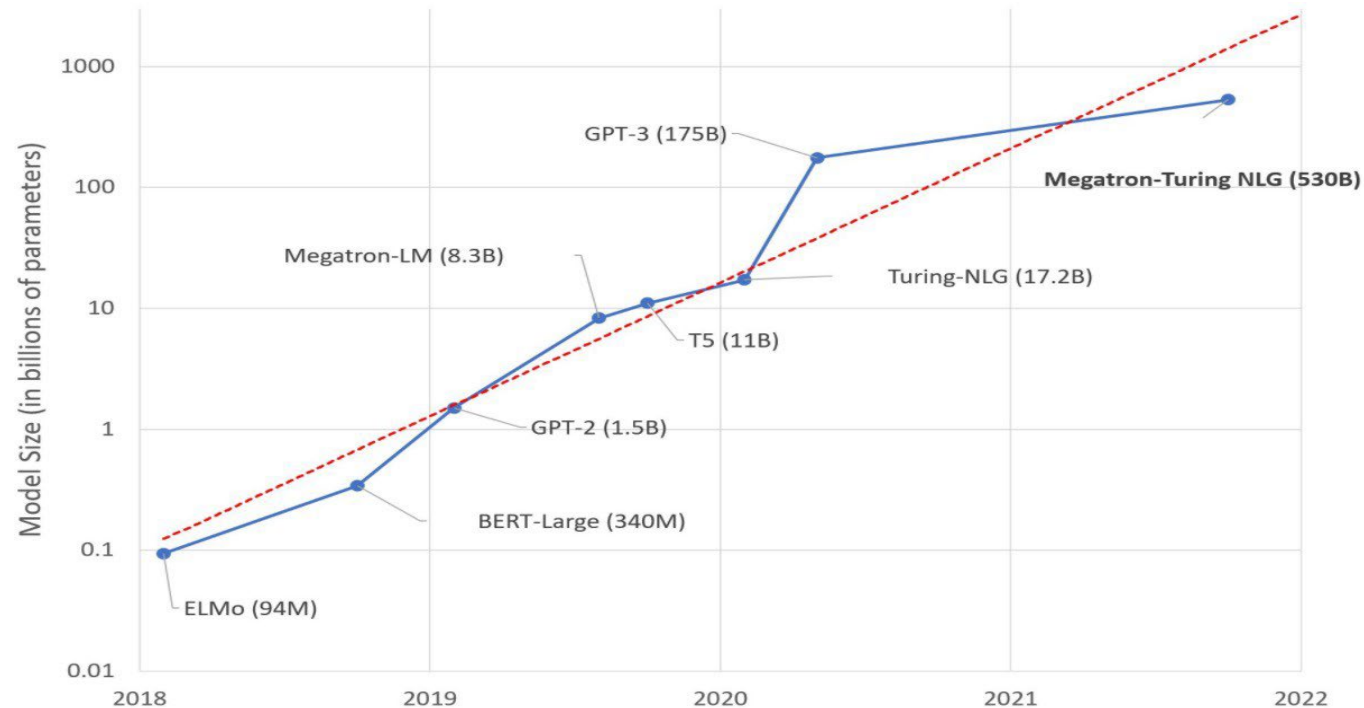


Photo credit: <https://www.microsoft.com/en-us/research/blog/using-deepspeed-and-megatron-to-train-megatron-turing-nlg-530b-the-worlds-largest-and-most-powerful-generative-language-model/>



# Large Language Models

- Not only they improved performance on many NLP tasks, but exhibited new capabilities



# Transformers - Summary

- Self-attention + positional embedding + others = NLP go brr
- Much faster to train than any previous architectures, much easier to scale
- Perform on par or better than previous RNN based models
  - Ease of scaling allows to extract much better performance